AC COMPENSATION OF AC SERVOMECHANISMS

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by

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Submitted in partial fulfillment
of the requirements
for the degree of
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PREFACE

The work on this thesis was performed at the United States Naval Postgraduate School at Monterey, California during the period November, 1953 to April, 1954.

The topic suggested itself after having had a course in Servomechanisms taught by Professor George J. Thaler of the Electrical Engineering Department of the Postgraduate School.

The author wishes to acknowledge the invaluable aid extended at all times by Professor Thaler during the preparation of this thesis.

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TABLE OF SYMBOLS AND ABBREVIATIONS

DC	Direct Current
AC	Alternating Current
E	Electric Potential
T	Torque
t	Time
ε	Error Signal, $\theta_i - \theta_o$.
θ	Angular Displacement, Radians
θ_i	System Input
θ_o	System Output
f	Coefficient of Viscuous Damping
J	Polar Moment of Inertia
$A_{1,2}$	Constants
$K_{1,2}$	Constants
K	One thousand Ohms
M	One Million Ohms
ϵ	Base of Naperian Logarithms
M_p	Maximum or Peak Overshoot
$\mathcal{L}$	Laplace Transform of
ω	Angular Velocity, Radians/second
ω_c	Angular Velocity, Carrier Frequency
ω_d	Angular Velocity, Data Frequency
λ, φ	Constant Angles
C	Capacitor
R, R1	Resistance

s	Laplacian operator
e_i	Input Voltage
e_o	Output Voltage
τ	Time Constant
cps	Cycles per Second
VLE	Velocity Lag Error
rpm	Revolutions per Minute
j	$\sqrt{-1}$
uf	Microfarads
uuf	Micromicrofarads
db	Decibels

SUMMARY

Objective:

To study the effect of various AC compensators on the performance of an AC positioning servomechanism.

General Methods:

An AC servomechanism was constructed such that its resonant frequency and the amount of underdamping could be varied over a considerable range by means of amplifier gain control. This servomechanism was used to determine the effect of various compensators on the transient and frequency responses of the system. These compensators were built in the laboratory and consisted of filter networks and vacuum tube circuits.

Conclusions:

It has been shown that a phase-lead compensator which is a considerable improvement over presently used AC compensators can be built utilizing the parallel T circuit and a preamplifier stage.

It has been indicated that it should be possible to construct a phase-lag compensator using more than one parallel T filter. This was unsuccessfully attempted using two parallel T circuits which were displaced from the carrier frequency.

It has been shown that cascaded high and low pass filters will not compensate the system over the range of data frequencies encountered in practice.

I

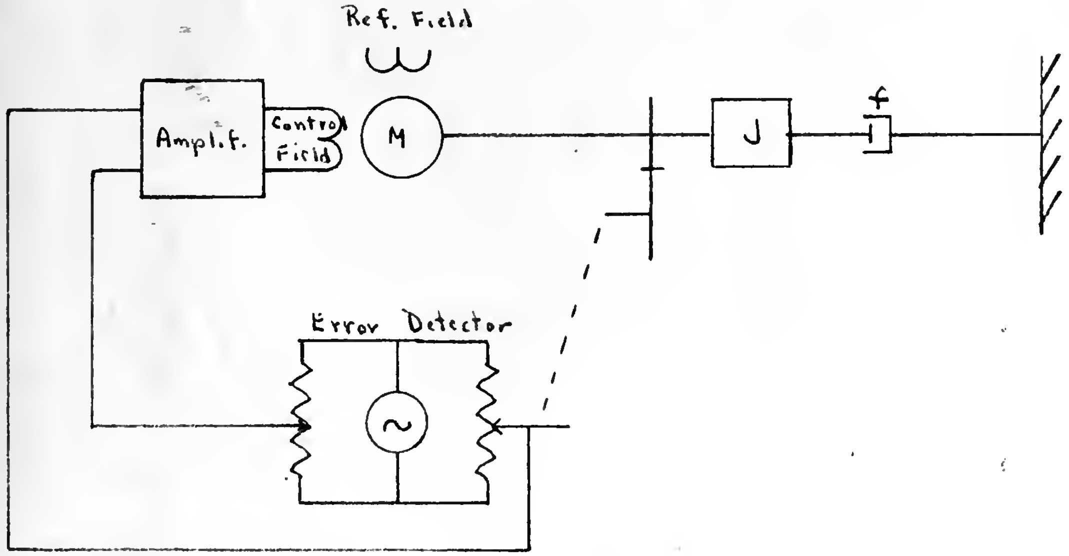
STATEMENT OF THE PROBLEM

This thesis is concerned with the problem of improving the performance, both transient and steady state, of an AC Servomechanism.

The word "servomechanism", as used in the material which follows, is defined as a closed loop automatic control system which specifially controls an output position. An AC servomechanism is a servomechanism in which not only is the error signal an AC signal, but the drive motor is an AC motor.

The primary purpose of any servomechanism is to drive an output load in such a manner that its position at all times corresponds to that of the input member of the system. However, any practical servomechanism can only approach this ideal due to the effects of inertia and friction.

An example of a simple closed loop servomechanism is as follows:



Writing the differential equation of the system:

$$T = J\frac{d^2\theta_o}{dt^2} + f\frac{d\theta_o}{dt}$$

Assuming:

$$T = K\varepsilon$$

then, $$K\varepsilon = J\frac{d^2\theta_o}{dt^2} + f\frac{d\theta_o}{dt}$$

but, $$\varepsilon = \theta_i - \theta_o$$

then, $$\frac{d^2\theta_o}{dt^2} + \frac{f}{J}\frac{d\theta_o}{dt} + K\theta_o = K\theta_i$$

This equation has for a general solution:

$$\theta_o = A_1 \epsilon^{\left(-\frac{f}{2J} + \sqrt{\frac{f^2}{4J^2} - \frac{K}{J}}\right)t} + A_2 \epsilon^{\left(-\frac{f}{2J} - \sqrt{\frac{f^2}{4J^2} - \frac{K}{J}}\right)t}$$

And depending on the parameters for a particular system, the system may be overdamped, critically damped or underdamped.

In considering the foreging, one is naturally led to the performance criteria for servomechanisms (both steady state and transient).

The most important steady state criterion of performance in a position servomechanism is, of course, accuracy. If the input signal requires that a given load move to a specific position, then the servomechanism is said to have perfect steady state performance if the commanded position is exactly obtained. However, few systems are perfect, and the accuracy of the sys-

tem is expressed in terms of the deviation of the output from the commanded position. The maximum error which can be allowed in any application is usually specified by the user.

In any given servomechanism, the transient performance requirements (transient performance refers to the time interval between the instant of command and the instant when the load reaches steady state) are more varied than the steady state requirements, but are equally important. A criterion of great importance in considering transient behavior is stability of the system. Because of the feedback loop, it is entirely possible for the system to hunt continuously (i. e., be unstable) if the system has been improperly designed. It is to be noted that a stable system will also oscillate if it is not in proper adjustment, but the preceding statement refers to a servomechanism which is impossible to stabilize by adjustment. With but few exceptions, specifications require that a servomechanism be stable.

Speed of Response. Generally, specifications for various types of servomechanisms permit several concepts of response speed.
They may be summarized as follows:

(a) In some systems the transient response is considered to be over only when the commanded steady state position has been obtained.

(b) In certain applications no overshoot of the response can be allowed.

(c) In the majority of cases the transient response is considered to be over when the output is within specified tolerances of the commanded position.

For requirements (a) or (b) the critically damped condition provides

the fastest response without overshoot and would probably be used. When a critically damped system is used, the speed of response is customarily expressed in terms of time constants -- four time constants is the usual figure of merit since over 98% of the change is made in this period.

Requirements as in (c) above are usually specified when it is vital that the output reach the commanded position (within the specified tolerance) in the shortest time. This is accomplished by the use of an underdamped system. However, when an underdamped system is used it is not possible to define the speed of response in terms of time constants; it depends not only on the specific system and the amount of underdamping, but also on the tolerance limits specified. Design of a system to obtain a specified response is based upon figures of merit that have developed from experience. Two of these figures of merit are: (1) Peak Overshoot, and (2) Frequency of the transient oscillation.

In an underdamped system the output will overshoot the desired steady state condition and a transient oscillation occurs. The first overshoot is always the greatest and its maximum value is termed the "peak" or "maximum" overshoot. Any undesirable features which result from overshooting will, therefore, result in major part from the first overshoot and the succeeding overshoots will be of lesser importance. It thus becomes important to know what undesirable features result from overshooting and the limitations which must be placed on the peak overshoot.

The two basic objections to large overshoot are: (1) The possibility of damage to the system which may result from the large accelerations inherent in a system with a large peak overshoot, and (2) The slowness with which a badly underdamped system approaches steady state. Experience has

indicated that satisfactory performance is more frequently obtained when the peak overshoot is limited to 1.5. That is, for a command signal of unit amplitude, the maximum overshoot would have a magnitude of 1.5.

The frequency of the transient oscillation is important for a number of reasons; chief among them are: (1) The system with the highest oscillating frequency will normally have the fastest response, and (2) If the oscillating frequency is at or near the natural frequency of some part of the system, undesirable resonance conditions may occur.

1. Frequency Response Methods for Analysis of Servomechanism Performance -

As has been indicated previously, it is possible to analyze any given servomechanism through the differential equations of the system, but for relatively simple systems the method becomes extremely laborious. Other methods have been developed for servo analysis which considerably reduce the labor involved. The method which will be used in the remainder of this discussion is the frequency response method.

From the discussion of performance criteria, it is apparent that the important feature of servomechanism performance is the response of the system as a function of time, when a specific input is applied. The mathematical work of Fourier has shown that the time response of a system is information which is contained implicitly in the frequency response. That is, if the frequency response is known for all frequencies from zero to infinity, it should be possible to determine what the transient response of the system will be for any specific input. However, mathematical methods for determining the transient response from the frequency response are quite involved. Simpler methods have been developed and are known as transfer function methods.

Rough correlations exist between the frequency response curves and the

transient response curves. They are:

(a) The peak overshoot of the transient is somewhat less than the height of the resonance peak of the frequency response.

(b) The transient oscillation frequency is approximately equal to the resonant frequency.

(c) Systems with high resonant frequencies usually have high response speeds, providing the height of the resonance peak is not excessive.

Powerful methods for the design of servomechanisms have been developed in recent years. These methods are primarily adaptions of methods used in designing feedback amplifiers in the electronic field. These methods involve plotting the transfer function of the proposed system. From the plot of the transfer function, the frequency response can be constructed and, hence, the transient response of the system predicted.

The transfer function is defined as:

$$\mathcal{L} \quad \frac{\theta_o}{\varepsilon}$$

The necessary procedure is to substitute in the transfer function equation a number of numerical values for the frequency and plot the results. The transfer function is, in general, a complex expression and the numbers obtained are vectors having a magnitude and phase angle. There are, therefore, three parameters to be considered in plotting: the frequency, the magnitude, and the phase angle. The transfer functions have been plotted in a number of ways; chief among them are:

(1) The Nyquist Diagram, in which the transfer function is plotted as vectors on polar coordinates.

(2) Logarithmic coordinates; that is, plot log magnitudes

versus log ω and phase angle versus log ω . This plot is sometimes known as the Bode Diagram.

The Nyquist Diagram permits ready determination of stability, resonance peak, and resonant frequency. However, the Bode Diagram permits ready determination of steady state performance and, moreover, shows the effects of important system adjustments and of adding system components. The height of the resonance peak and the natural frequency of the system may be estimated.

In practical design the Bode Diagram is most commonly used. Although the foregoing methods were initially developed for use in design work, no small part of their utility lies in the fact that the process can be reversed; that is, the frequency response of a given system is obtained and converted to the Bode Diagram plot so that the effect of system adjustments are more readily apparent. The Bode Diagram and associated diagrams will be used in this thesis to compare the performance of the system under different operating conditions.

2. Compensation of Servomechanisms

The specifications under which a servomechanism are constructed are often conflicting. That is, the specifications may demand, for example, that the velocity lag error with a constant velocity input be five degrees or less (the position of the output shall not be displaced more than 5 degrees from the position of the input command signal when the input signal is being applied at a specified rate.); the satisfaction of the foregoing may require that the gain be set so high as to result in a grossly underdamped system, and this would violate the specifications regarding the maximum peak overshoot. The condition just outlined would be

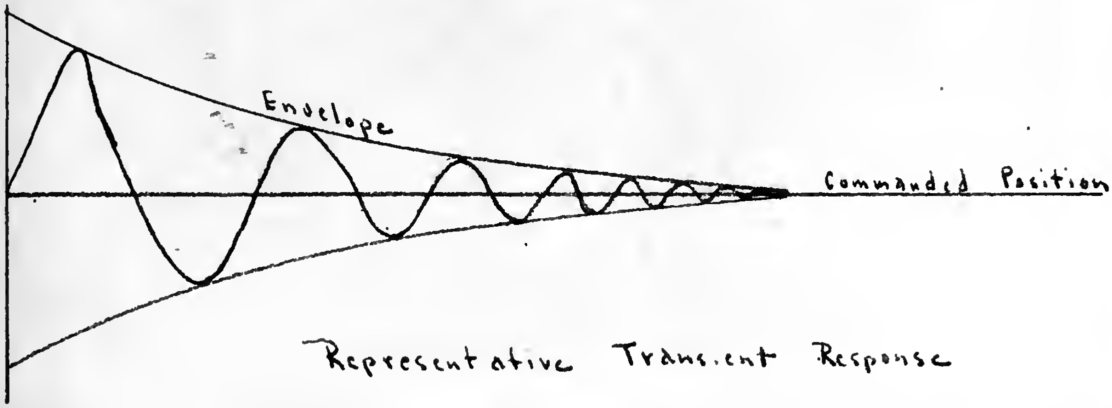
characterized on the Bode Diagram by a positive phase margin at the cross-over point which was too small.*

The problem of compensation, stated in its simplest terms, is that of modifying the transient response of the system so that both steady-state and transient specifications can be realized.

Compensation of servomechanisms can take many forms, but two of the most widely used methods of compensation (and the only methods which are considered in this thesis) are known as "phase-lead" and "phase-lag" compensation.

3. Phase - Lead Compensation

A clearer concept of phase-lead compensation can be gained by a consideration of the underdamped response of a servomechanism. From the graphical picture and the differential equations of the system it is apparent that the envelope of the transient response is a function of the coefficient of viscuous damping. The peak overshoot could be



Representative Transient Response

reduced by increasing the viscuous damping, but this produces an undesired increase in the velocity lag error. However, if the curve be examined, it will appear that a correction which is a function of the rate of change of the error could be used to modify the transient response while at the same

*See "Servomechanism Analysis" by Thaler and Brown for a discussion of design and analysis by use of the diagram.

time leaving the steady state response unchanged. That is, in addition to the error signal, a signal which is the time derivative of the error signal is applied to the motor.

Consider the following:

$$\varepsilon = \sin \omega t$$

then,

$$\frac{d\varepsilon}{dt} = \omega \cos \omega t$$

and $K_1 \varepsilon + K_2 \frac{d\varepsilon}{dt} = K_1 \sin \omega t + K_2 \omega \cos \omega t$

$$= \sqrt{K_1^2 + \omega^2 K_2^2} \sin (\omega t + \lambda) \quad ; \quad \lambda = \tan^{-1} \frac{\omega K_2}{K_1}$$

or,

$$K_1 \sqrt{1 + \omega^2 \left(\frac{K_2}{K_1}\right)^2} \sin (\omega t + \lambda)$$

Since the phase angle of the error signal has been shifted in the positive direction by the amount λ, this method of compensation has been termed "phase-lead" compensation. Equivalent terms are "Derivative" and "Error-Rate" compensation.

The effect of phase-lead compensation considered in relation to the Bode Diagram is to shift the phase angle curve upward (in the positive direction). It also has an effect on the magnitude curve, as can be seen from the above equations, but not a proportional effect. Hence, phase-lead compensation results in an increase in the positive phase margin at the crossover point.

4. Phase-Lag Compensation

Although phase-lead compensation tends to reduce velocity lag error,

it has no effect on steady state errors; for example, the steady state errors due to the effect of the load exerting a torque on the system. Consider the steady state condition of a servo with velocity input or a stationary positioning system with load torque. In either case there is a constant positional displacement between input and output--in other words, a constant error. The only way to reduce this error is to increase the drive torque. The derivative signal is a signal proportional to the rate of change of the error, and since the error is not changing, it would not be effective. However, if a signal proportional to the integral of the error were introduced, it is readily seen that the error must ultimately be reduced to zero because the integral term eventually becomes infinite if the error does not go to zero.

Consider the following:

$$\varepsilon = \sin \omega t$$

then

$$\int \varepsilon \, dt = \int \sin \omega t \, dt = -\frac{1}{\omega} \cos \omega t$$

and, $K_1 \varepsilon + K_2 \int \varepsilon \, dt = K_1 \sin \omega t - \frac{K}{\omega} \cos \omega t$

$$= \sqrt{K_1^2 + \frac{K_2^2}{\omega^2}} \sin(\omega t - \lambda) \quad ; \quad \lambda = \tan^{-1} \frac{K_2}{\omega K_1}$$

or,

$$K_1 \sqrt{1 + \left(\frac{K_2}{K_1}\right)^2 \frac{1}{\omega^2}} \sin(\omega t - \lambda)$$

The origin of the term "phase-lag" compensation is thus obvious from the above equation. It is to be noted in the above equation that, in theory, the signal should be infinite for a data angular frequency of

zero, and should become smaller as the frequency increases.

The effect of phase-lag compensation considered from the viewpoint of the Bode Diagram for the system is to bend both the magnitude and phase angle curves downward.

The problem with which this thesis is directly concerned is the study of various methods by which phase-lead and phase-lag compensation may be employed in improving the performance of an AC Servomechanism.

II

HISTORICAL DEVELOPMENT

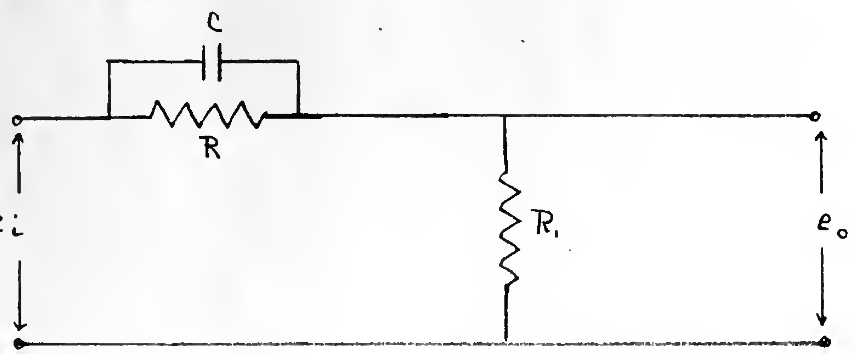
In the early history of servomechanisms many of the systems employed DC throughout; the error signal was a DC signal and the drive motor, error detectors, etc. were DC. On the other hand, many early systems also used synchros as error detectors although the drive motor was DC. The methods initially developed for servomechanism compensation were, nonetheless, networks designed to operate on a modulated DC signal.

As was indicated in Chapter I, compensation is necessary when it is not possible to meet both the steady-state and transient specifications. The most common types of compensation developed for DC servos were passive filter networks.

1. DC Phase-lead Compensation.

High pass filters were developed to approximate the requirements for phase lead or derivative compensation. In general they produce a lead in the phase angle of the error signal and also give greater attenuation at the low frequencies than at high frequencies. This will be recalled as the requirements for phase lead compensation from Chapter I.

The high pass filter most commonly used for DC compensation is as indicated below.



Utilizing Laplace Transforms, the transfer function for the above network is developed as follows:

$$e_o = \frac{e_i R_1}{R_1 + \dfrac{R/sC}{R + 1/sC}} = \frac{e_i R_1}{R_1 + \dfrac{R}{sCR+1}}$$

$$\frac{e_o}{e_i} = \frac{R_1(sCR+1)}{sCRR_1 + R_1 + R} = \frac{R_1}{R+R_1} \cdot \frac{sCR+1}{sCR\left(\frac{R_1}{R+R_1}\right)+1}$$

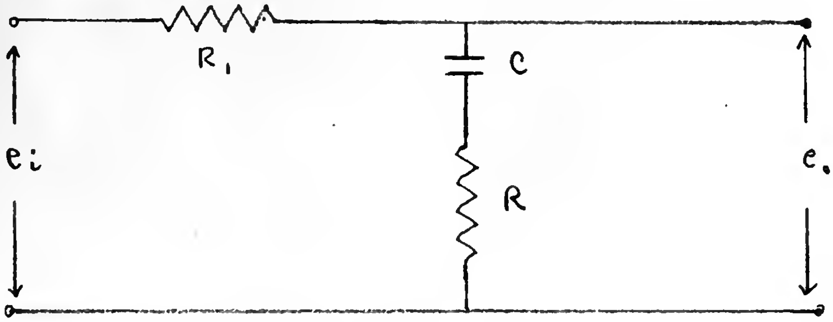
Defining: $RC = \tau$; $\dfrac{R_1}{R+R_1} = \alpha$

$$\frac{e_o}{e_i} = \alpha \frac{s\tau+1}{s\alpha\tau+1}$$

The effect of the above transfer function considered in relation to the Bode Diagram is to shift both the magnitude and phase angle curves upward for a definite range of frequencies governed by selection of the filter components. That is, the filter is not a true differentiator, but will produce an effective differentiation over a limited frequency range.

2. DC phase-lag compensation

In a similar manner effective phase lag compensation was accomplished by use of a low pass filter network. The most widely used network is shown below.



The development of the transfer function for the network follows:

$$e_o = \frac{e_i\,(R + 1/sc)}{R_1 + R + 1/sc} = \frac{e_i\,(SCR + 1)}{SC(R + R_1) + 1}$$

$$\text{or,}\quad \frac{e_o}{e_i} = \frac{S\tau_1 + 1}{S\tau_2 + 1}$$

$$\text{where}\quad \tau_1 = RC \quad \text{and} \quad \tau_2 = (R + R_1)\,C$$

The phase-lag compensator must give the converse of the phase lead compensator; it must produce a lag in the phase angle of the error signal and must pass the low frequency signals while attenuating the higher frequency signals.

The above network, like the lead network just developed, does not give a true integration, but effectively accomplishes the job over a limited range of frequencies governed by the filter parameters. The effect of the filter relative to the Bode Diagram is to shift both the magnitude and phase angle curves downward over the range of frequencies governed by the filter.

It is well at this stage to point out the fact that although a true integrator will shift the phase of the error signal, as will the approximate integrator above, the effect of the phase shift is actually unwanted. The desired effect is to decrease the magnitude of the signal with frequency. A reduction in the magnitude with no shift in phase will result in a lower resonant frequency (which, in itself, is not usually desirable);

whereas, the effect of a shift in phase in the lagging direction unaccompanied by a decrease in signal strength has the effect of making the system more oscillatory. For these reasons phase lag compensation by the use of filters is ordinarily designed so that it operates at frequencies well below the crossover point frequency. This, in effect, utilizes the decrease in signal magnitude feature of the filter, while allowing the phase to return to very nearly its original value at the crossover point.

3. AC compensation

With the foregoing background in compensation, it was natural to consider compensation of the first AC servomechanisms by the use of DC filter networks. Before compensation of the error signal could be accomplished by DC networks it was necessary, of course, to rectify the modulated AC error signal, and after compensation to modulate the carrier with the corrected signal.

Although the DC compensation of AC servos has been successful, it has disadvantages in that it requires additional components--expensive components, when the better circuits are used.

To overcome the disadvantages of DC compensation, much time and effort has been spent with more or less success. Inasmuch as the subject of this thesis is the study of various AC compensation methods for AC servomechanisms, some of the successful approaches to AC compensation will be subsequently discussed.

III

BASIC SERVOMECHANISM

1. Design of Basic Servo

When the thesis topic was first chosen, it was decided that the best way of testing the proposed compensators was by using them in conjunction with an actual AC servomechanism.

In accordance with the foregoing, the first project in connection with the thesis was the construction of an AC servomechanism. The use of the servo to test both phase-lead and phase-lag compensators governed the specifications for the system. The specifications were not rigid, of course, but were such as would result in a servo which would operate at a resonant frequency of ten cps or higher. Further, the desirability of linear operation is obvious.

The figure ten cps for a resonant frequency was chosen so that compensation by phase-lag methods (which invariably result in a lowering of the resonant frequency) would not result in a resonant frequency lower than about four cps. This figure was desirable in view of the fact that the available test equipment would not provide a complete frequency response for resonant frequencies much lower than four cps.

The choice of ten cps for a resonant frequency for the system did not mean that the system was required to have a peak overshoot less than 1.5 in addition to the resonant frequency requirements. A peak overshoot of as high as four or five was felt to be satisfactory, since it was reasonable to expect that this could be reduced to 1.5 or lower by phase-lag compensation with a consequent reduction in resonant frequency of six cps or less -- i. e., a resultant resonant frequency of four cps or

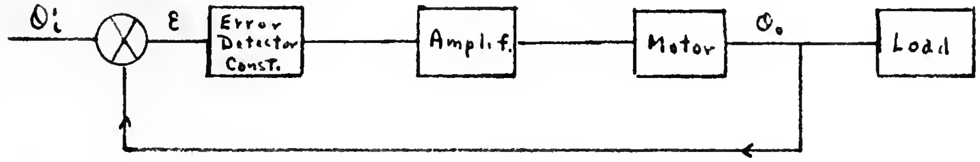
greater.

These restrictions on the resonant frequency very definitely governed the characteristics of the components selected for the system. Although the selection of a resonant frequency of 10 cps is not by any means an impossible requirement, the fact that the components cannot be carelessly selected is made evident when one considers that the average servomechanism built from readily available laboratory parts with the only selection being that the parts fit will result in a system with a resonant frequency of 4 to 5 cps.

With the above in mind, components were chosen having low moments of inertia and minimum friction. Since the most common AC servomechanisms utilize 2 phase motors, a Kearfott R111-2A two phase 400 cycle drive motor was decided upon and the system was built around this motor. (Actually a larger motor was first tried, but tests indicated that its moment of inertia was too high). Synchros were chosen as error detectors because of their very low friction. For the necessary gearing between units, very small lightweight commercial gears were utilized and gear ratios were chosen which would minimize the effects of load and gear train inertia insofar as practicable.*

An AC power amplifier which had been built for a previous servomechanism thesis was tested, found suitable for this application, and was used without change.

A block diagram of the basic system appears below.



*See "Theory of Servomechanisms" by James, Nichols, and Phillips for a discussion of optimum gear ratios.

Transfer functions for the individual blocks and system are as follows:

Error detector: K_s

Power amplifier: K

Motor: $\dfrac{K_m}{s(s\tau_m + 1)}$ *

System transfer function: $\dfrac{KK_sK_m}{s(s\tau_m+1)}$

The construction of the servomechanism in accordance with the foregoing resulted in a servomechanism which could be operated at a resonant frequency of from 3 cps to a maximum of 12 cps with a peak overshoot of 4 before the system became unstable. This variation in resonant frequency could be smoothly controlled by variation in the power amplifier gain.

Complete diagrams of the system, power amplifier, and identification of components, gear ratios, etc. are included in the appendix.

2. Response of the Uncompensated System

There are several methods for determining the effectiveness of compensators, but two which are in general use are described in subsequent paragraphs.

In the first, the frequency response of the system is obtained for a gain setting which results in a desirable resonant frequency and peak overshoot (In this instance a peak overshoot of between 1.5 and 1.9 was chosen). For this condition the velocity lag error is determined. This frequency response and velocity lag error is then used as a standard.

A check of the effectiveness of any particular compensator is made by inserting the compensator in the circuit and adjusting the system gain

*See "Principles of Servomechanisms" by Brown and Campbell for a development of the transfer function for a two phase motor.

until the peak overshoot and resonant frequency are as close to the standard as possible. When this condition is obtained, the velocity lag error is again checked. The effectiveness of any particular compensator will be reflected in the amount of reduction of the velocity lag error.

The foregoing is in opposition to the more usual procedure of driving the system until it is grossly underdamped, then judging the effect of the compensator by the improvement in the frequency response. However, the use of VLE (Velocity Lag Error) as one criterion for effectiveness was selected because the examination of phase-lag compensators was envisioned as well as phase-lead compensators, and the real test of effectiveness for a phase-lag compensator lies in the reduction of velocity lag error.

Both methods will be used in this thesis, however, and whichever criterion seems to be the better for any particular compensator will be the one used. In some cases both will be used.

The device used to obtain the transient and frequency responses was the Dynamic Analyzer manufactured by the Industrial Control Co. The device furnishes either a step or a sine function to drive the servo along with a means for determing phase displacement between the input and output signals. In some instances the analyzer in conjunction with an oscilloscope was used to determine the frequency response, while in others the input and output signals were fed to a Brush Recorder and the frequency response determined from the recorder trace.

The frequency response of the uncompensated system had a resonant frequency of 6 cps, a peak overshoot of 1.8, and a VLE of 16.6° at 250 rpm. The Bode diagram for the uncompensated system is included in the appendix

as is the schematic diagram of the test set-up.

3. D. C. Compensated System

D. C. compensation of AC servomechanisms has been widely and effectively used as has been indicated previously. For this reason, it was felt that the effect of DC compensation on the basic servo could be used advantageously as a basis for judging the performance of the AC compensators.

Before DC compensation could be utilized, however, it was necessary to construct rectifier and modulator circuits. To accomplish this, choppers were used in conjunction with a phase shifter. The purpose of the phase shifter being to shift the phase of the operating coil voltage until it was in phase with the signal voltage. A synchro generator was used for this purpose, but due to the fact that only two phase 400 cycle power was available, it was necessary to use Scott transformers to convert to three phase. Circuit diagrams are given in the appendix.

Rather than design a DC filter network to compensate the system, a Krohn-Hite compensator was used, which is a filter of the type previously described except that the filter parameters are variable, allowing smooth adjustment of the frequency range of the compensation.

The filter parameters were adjusted so that phase-lag compensation was effective from .05 to .6 cps, resulting in a resonant frequency of 4.5, a peak overshoot of 1.7, and a velocity lag error of 2.7 degrees at 250 rpm. The Bode Diagram for the compensated system appears in the appendix.

IV

THE PARALLEL T

It will be recalled from chapter I that for an error signal of the form

$$\varepsilon = K \sin \omega t$$

the resultant when a derivative signal is added to the error signal is

$$K_1 \varepsilon + K_2 \frac{d\varepsilon}{dt} = K_1 \sqrt{1 + \omega^2 \left(\frac{K_2}{K_1}\right)^2} \sin(\omega t + \lambda)$$

It is to be noted at this point that the ω in the above equation refers to the angular frequency of the data signal and not to the angular frequency of the carrier. In DC error detectors, a constant error produces a constant error voltage, while a pulsating or alternating error produces a pulsating or alternating voltage of the same frequency. On the other hand, with an AC error detector, a constant error produces an alternating voltage.

$$\varepsilon = K \cos \omega_c t$$

where ω_c refers to the carrier angular frequency, and an alternating error of frequency ω_d produces an alternating voltage

$$\varepsilon = K \cos \omega_d t \cos \omega_c t$$

In other words, an amplitude modulated carrier. (Obviously, sines could have been used as well as cosines, but the more usual development in the literature utilizes the cosine form as will subsequent developments in this thesis).

By the use of trigonometric identities the above may be expressed in the

usual sideband notation as

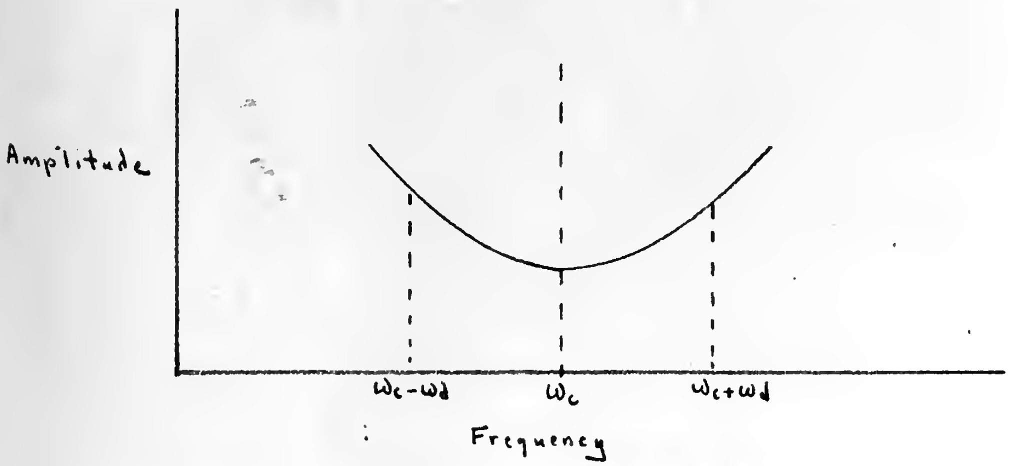
$$\mathcal{E} = \frac{K}{2}\left[\cos(\omega_c + \omega_d)t + \cos(\omega_c - \omega_d)t\right]$$

For example, if in a given servo the data frequency varies from zero to 15 cps, then the error voltage frequency will vary between 385 and 415 cps for a system with a carrier frequency of 400 cps.

From the foregoing discussion, it will be readily apparent that the compensating network must be such that at the carrier frequency, ω_c , the voltage passed is equal to K_1, and at frequencies differing from ω_c by the modulation frequency, ω_d , it must pass a voltage equal to

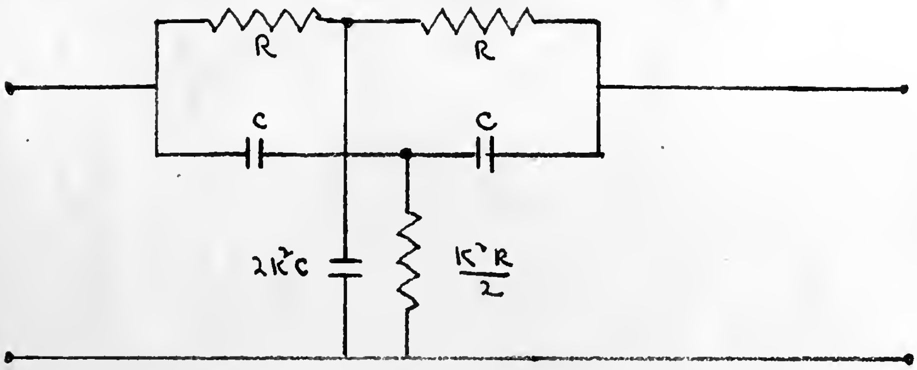
$$K_1\sqrt{1 + \omega_d^2\left(\frac{K_2}{K_1}\right)^2}$$

and leading the phase of the error signal by an angle equal to $\tan^{-1}\frac{K_2\,\omega_d}{K_1}$. Such a frequency response is shown graphically below.



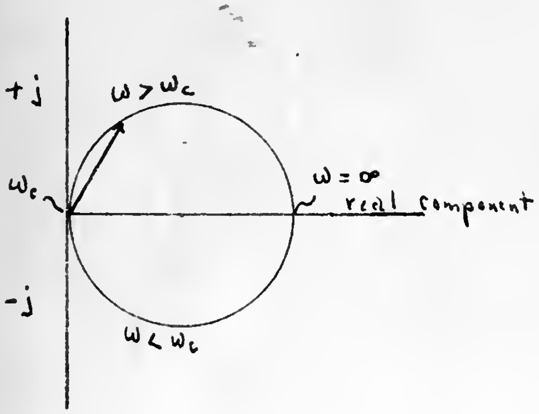
A network which will produce the above frequency response and which has been widely used is the Parallel T, first described by Augustadt and used in power supply circuits. Many papers have appeared in the literature on the parallel T and commercial servo amplifiers are available which utilize the

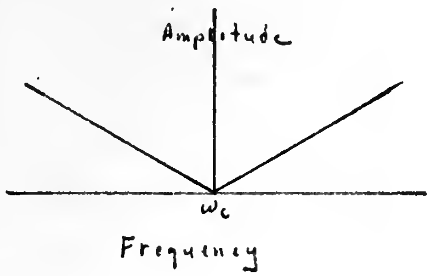
circuit. The basic circuit appears below.



The value of K for which minimum attenuation is obtained for a given bandwidth is equal to unity.

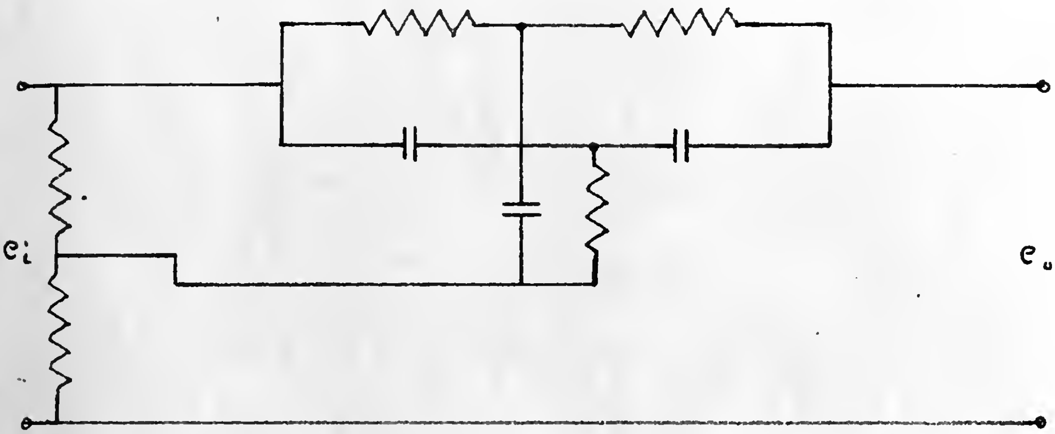
The parallel T may be analyzed by replacing the circuit with its equivalent lattice*, but it can be done equally well by using mesh current methods. The result of this analysis is that for frequencies close to the null frequency the phase of the input signal has been shifted by ninety degrees--plus 90° for frequencies above the null frequency and minus 90° for frequencies below. The graphical and frequency response of the filter is shown below.





*See "Servomechanism Fundamentals" by Lauer, Lesnick, and Matson.

From the above response it is apparent that a bypass channel must be provided before the filter can be used as a phase lead compensator. The modified filter, suitable for use as a phase-lead compensator, is sketched below.



Although a laborious procedure, this circuit can be analyzed by ordinary methods and a transfer function in terms of the data frequency can be written. However, during the course of the thesis, I developed a method of analysis that utilized the known response of the parallel T and which gives the desired data transfer function by relatively simple methods. The method can best be explained by means of an example. The circuit above, with a known transfer function, is used as the example.

Assuming input error signal equal to

$$\varepsilon = E \cos \omega_d t \cos \omega_c t$$

or,

$$E/2 \cos (\omega_c + \omega_d)t + E/2 \cos (\omega_c - \omega_d)t$$

With the above input, the output is:

$$j \frac{K_1}{2} \cos (\omega_c + \omega_d)t - j K_1/2 \cos (\omega_c - \omega_d)t$$

but,

$$j \cos \alpha t = - \sin \alpha t$$

Therefore, the output may be expressed as

$$-\frac{K_1}{2}\sin(\omega_c+\omega_d)t \quad + \frac{K_1}{2}\sin(\omega_c-\omega_d)t$$

Assuming a bypassed signal of equal amplitude, the sum of the bypassed and parallel T output signals is

$$\frac{E}{2}\cos(\omega_c+\omega_d)t \quad - \frac{K_1}{2}\sin(\omega_c+\omega_d)t$$

$$+\frac{E}{2}\cos(\omega_c-\omega_d)t \quad + \frac{K_1}{2}\sin(\omega_c-\omega_d)t$$

Multiplying and dividing by $\frac{1}{2}\sqrt{E^2+K_1^2}$, the above may be expressed as

$$\frac{1}{2}\sqrt{E^2+K_1^2}\,\Big[\cos\varphi\cos(\omega_c+\omega_d)t \quad - \sin\varphi\sin(\omega_c+\omega_d)t$$

$$+ \cos\varphi\cos(\omega_c-\omega_d)t \quad + \sin\varphi\sin(\omega_c-\omega_d)t\Big]$$

but K_1 is a function of $\omega_d \equiv K\omega_d$.

By trigonometric identities the above becomes

$$\frac{1}{2}\sqrt{E^2+(K\omega_d)^2}\,\Big[\cos\left(\omega_c t+\{\omega_d t+\varphi\}\right)+\cos\left(\omega_c t-\{\omega_d t+\varphi\}\right)\Big]$$

$$=\sqrt{E^2+(K\omega_d)^2}\,\cos(\omega_d t+\varphi)\cos\omega_c t$$

Where $\quad \varphi = \tan^{-1}\dfrac{K\omega_d}{E}$

In transfer function notation this may be written

$$E(s\tau+1) \qquad \text{where} \quad \tau = K/E$$

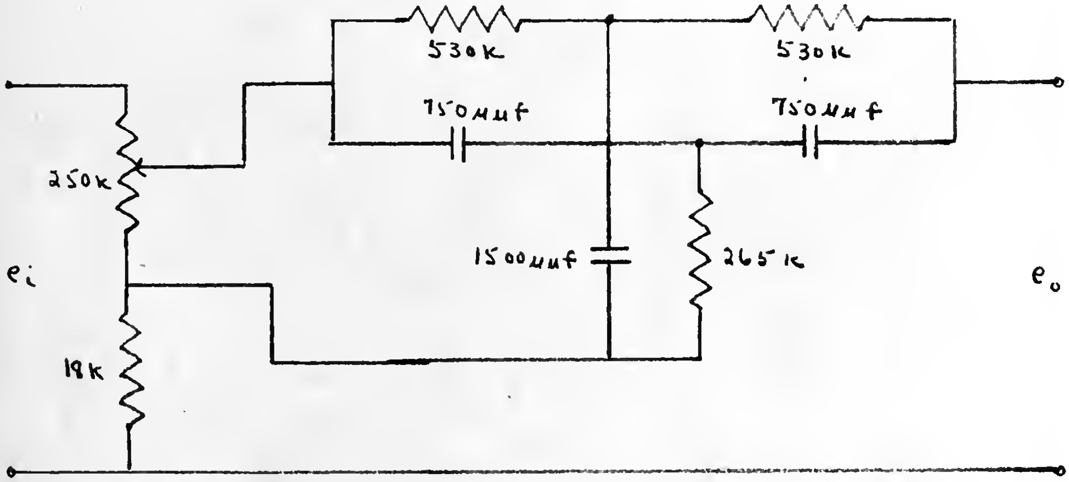
If the above method were restricted to the above circuit, it would be

of limited interest since the transfer function for the circuit is known. However, the method can readily be used on any circuit which employs the parallel T as part of the circuit.

V

AN IMPROVED PHASE-LEAD COMPENSATOR USING THE PARALLEL T

It was pointed out in the previous chapter that commercial servo amplifiers which use the parallel T circuit are in common use. The circuit employed in one of the commercial amplifiers is given below.



The above circuit is essentially the same as the circuit shown before except for the fact that values for the filter components are added and that a potentiometer input to the filter is used.

The potentiometer input is obviously for the purpose of controlling the amount of compensation. For maximum effect about 93% of the error signal is taken as an input to the parallel T and 7% of the signal is by-passed. From the frequency response of the commercial compensator which appears in the appendix it is apparent for error frequencies of the order of 16 cps, the signal through the compensator has a maximum value of .147 volts with a maximum phase shift of about ten degrees.

The above analysis was made on the basis of the frequency response shown in the appendix; however, frequency responses were obtained for a

number of commercial filters and the one shown is representative. From the foregoing it is apparent that the above circuit configuration will have a negligible effect as far as phase-lead compensation is concerned, and from my limited experience I have found this substantiated in practice. Further, not much can be done to improve the performance by changing the ratio of voltage input to the filter. Further reduction of the bypassed signal must be compensated by an increase in the amplifier gain with attendant noise and pick-up difficulties. (Any circuit using a null filter must be carefully shielded and grounded since the circuit offers very high input impedance at the null frequency and much lower impedance for signals of different frequency, such as would be induced by stray fields.)

It was early decided in the consideration of this problem that it would be necessary to bypass a signal of greater amplitude. However, if this were done it would be necessary to raise the voltage level of the output signal of the filter before the two signals were added. This, in itself, offers no problem, and the construction of the compensator proceeded along these lines.

A two stage preamplifier was constructed utilizing a twin-triode 6SN7. The two stages shift the phase of the filter output by 360° and, therefore, the previous mathematical development is still valid. A parallel T to be used in conjunction with the preamplifier was built of quality components carefully measured by the most precise bridges available. The figure of 723 uuf was determined by measuring a large number of capacitors and selecting two which were precisely equal.

The greatest difficulty encountered in this approach to the problem lay in the addition of the amplified filter output and the bypassed signal.

In the conventional circuit no difficulty arises because one side of the input is grounded and this side of the signal is the input to the grounded side of the power amplifier. However, in the proposed circuit the filter output is to be amplified, and since a single power supply seems desirable, it is necessary to ground one side of the filter. Opposed to this is the mandatory requirement that the input signal be grounded at the point of lowest potential in order to reduce the problem of pick-up. This particular difficulty is readily overcome by using an isolating transformer for the bypass signal. The most obvious method for adding the two signals was to put the secondary of the bypass transformer in series with the output of the preamplifier. Although not apparent initially, this solution resulted in positive feedback and resultant instability of the amplifier. Several approaches to the problem were made, but it was eventually solved by applying the filter output and bypassed voltages to the inputs of two cathode followers operating in push-pull. The outputs of the cathode followers were added in the primary of a transformer, and the transformer secondary voltage was applied to the input of the power amplifier.

The complete circuit diagram for the filter, preamplifier, and cathode followers is included in the appendix.

Reference to the frequency response for the parallel T and preamplifier in the appendix will show that the response of the filter and preamplifier is very much better than the response of the commercial compensator. At a high preamplifier gain it is possible to double the input signal to the power amplifier when the error frequency goes from zero to twenty cycles per second. This increase in signal amplitude is accompanied by a shift in phase of over 50° at twenty cps. Moreover, by means of the

preamplifier gain control, the amount of compensation can be smoothly varied from zero to the maximum value. This is to be compared to the response of the commercial amplifier which gave an increase in magnitude of less than three percent and a phase shift of about ten degrees for the same error frequency.

The frequency response for the filter and preamplifier was obtained by using the servomechanism analyzer for an amplitude modulated input signal, which furnishes modulation frequencies from zero to about twenty cps and by chopping the input and output signals which were then applied to a two channel Brush Recorder. This method permits a rapid evaluation of any particular compensator and, further, gives a permanent record. A circuit diagram of the test set-up used is included in the appendix.

To investigate the effect of the compensator on system performance, frequency responses were taken for three separate power amplifier gain levels, giving three degrees of underdamping. Then without change in the gain of the power amplifier, but with a step input rather than a sine, the gain of the preamplifier was adjusted to give a more satisfactory transient response. Following this step, frequency responses for the system were again taken.

The Brush Recorder trace of the transient responses for two power amplifier gain levels (both uncompensated and compensated) are included in the appendix as is the Bode Diagram for one gain level. Although the most striking evidence of the effectiveness of the compensator appears in the transient response comparisons (particularly the high gain setting), it is apparent in the Bode Diagram as may be seen in the change in the slope of the magnitude curve from about -12 db/octave to -6 db/octave. This change

in the magnitude curve is accompanied by a change in the phase angle curve. The phase angle curve decreases at a much slower rate and becomes a minimum just before the crossover point in the compensated response; whereas in the uncompensated response, the phase angle curve continually decreases and approaches -180° as a limit.

The system transient and frequency responses were obtained in a manner similar to that for the frequency response of the compensator; i. e., the input and output signals were chopped and applied to a two channel Brush Recorder. A simplified schematic of the test circuit is in the appendix.

To summarize, I think it is evident that this method of phase-lead compensation is much superior to the method commonly used at present. And economically speaking, the cost of the filter, preamplifier, and power amplifier will be no more than the cost of the present filter and high gain power amplifier, except for the fact that two additional transformers are used. However, when one compares the performance of the two compensators, the increased cost of the additional transformers seems to be well justified.

A low w versus magnitude and phase angle diagram for both the commercial compensator and the parallel T-preamplifier circuit are included in the appendix.

VI

PHASE LAG COMPENSATION USING TWO PARALLEL T CIRCUITS

It will be recalled from the discussion of phase-lag or integral compensation in chapter I that for a sine function error signal, the output signal voltage of the compensator added to the error signal results in a signal voltage of the following form:

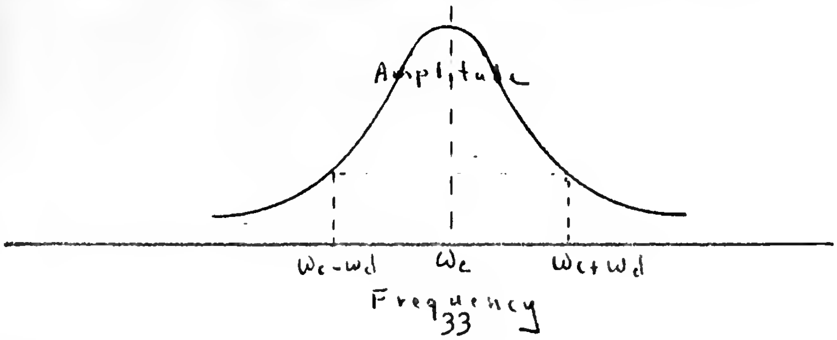
$$K_1 \varepsilon + K_2 \int \varepsilon \, dt = K_1 \sqrt{1 + \left(\frac{K_2}{K_1}\right)^2 \frac{1}{\omega^2}} \sin(\omega t - \lambda)$$

or, equally well

$$K_1 \sqrt{1 + \left(\frac{K_2}{K_1}\right)^2 \frac{1}{\omega^2}} \cos(\omega t - \lambda)$$

Where, as before, the ω refers to the data angular frequency. In other words, the signal output approaches infinity as the frequency of the error signal approaches zero and becomes less and less as the error signal frequency increases.

It will further be recalled from the discussion in chapter II that although a true integrator will shift the phase of the signal in a negative direction, this effect is actually unwanted. The desirable output of a phase-lag compensator is a signal which decreases as a function of frequency, but in which the phase angle of the signal is unaffected by frequency. Such a response is shown graphically below.



Although phase-lag compensation can be accomplished by rectification and DC filter networks, as previously indicated, the advantages of an AC network are obvious. One solution to the problem has been to use a tuned circuit which will give the above response; however, the tuned circuit has many disadvantages and is not generally used*. When AC networks are considered, the desirability of an RC filter from the standpoint of component size, weight, and economy is evident.

Considering the above, the possibility of using a Parallel T in a phase-lag circuit immediately suggests itself. The major problem, of course, being the inversion of the output signal. In the light of the previous work done on this thesis, one is naturally led to considering modifications of the circuit used to phase-lead compensate the system. If the bypass feature of the previous compensator is retained, then the problem is to subtract the output of the filter from the bypassed signal. However, for data frequencies of the order of 15 to 20 cps, the output of the parallel T is essentially in quadrature with the bypassed signal. And since the subtraction (or addition) of any signal in quadrature with the bypassed signal can only result in a signal of greater magnitude, one is led to a consideration of the effect of shifting the phase of the filter output. Moreover, if phase-shift of compensator output is undesirable, the filter output must be shifted by 90°; this will result in a signal either in phase or 180° out of phase with the bypassed signal, depending on whether the filter output is plus 90° or minus 90° with reference to the bypassed signal. This is precisely the reason why one parallel T cannot be used. It will be recalled that the input to the filter is composed of the two sideband signals for ω_d not zero and the output likewise contains both signals, one shifted

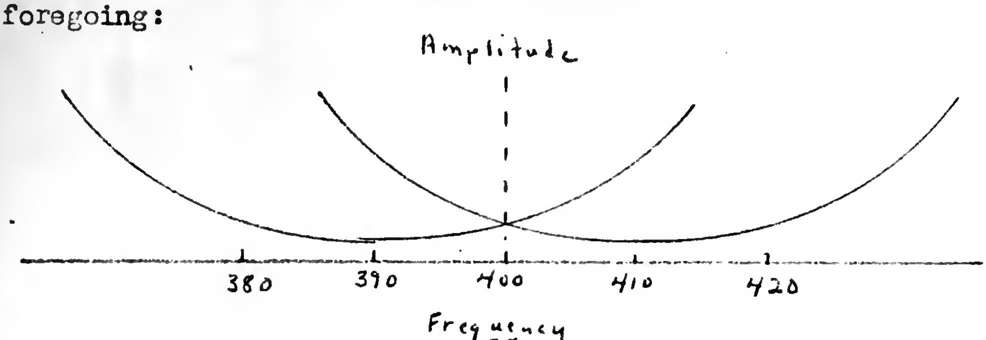
*See "Servomechanism Fundamentals" by Lauer, Lesnick, and Matson.

by plus 90° and the other by minus 90°. A solution to the problem would be to invert the minus 90° sideband signal and to shift the phase of the two in-phase signals by plus 90°; but when one realizes the difficulties connected with such selective inversion, other solutions to the problem are sought. Having eliminated the single T, the feasibility of using more than one parallel T to accomplish the same result was considered.

If the output of one filter were such as to be predominantly plus 90°, and the output of a second filter were predominantly minus 90°, then the output of the two filters could be subtracted and the resultant signal (which would be in phase) could be used as an input to a phase shifting circuit. As before, the output of the filters would have to be amplified before being added to bypassed signal; therefore an amplifier which also shifted the phase of the signal would be desirable. This can be done by adjusting the value of the coupling capacitance between stages of an audio amplifier. To convert the amplifier used previously, it would only be necessary to replace the 0.02 coupling capacitors with capacitors whose capacitive reactance was equal to the grid resistor, R_g.

The possibility of constructing an asymmetrical parallel T was considered (output on one side of the null point greater than the other side), but it seemed the easier solution was displacement of the null point (see frequency response of the commercial parallel T in the appendix).

The following is a graphical illustration of what is suggested by the foregoing:



For an in-phase input to the two filters and with the outputs in opposition, the filter centered at 390 cps would give a high output for the positive sideband and the other a high output for the negative sideband. This is the desired result, and as a check on the physical reasoning, a mathematical analysis of the circuit was made. It follows: (Reference to the circuit diagram for the two filters and phase-shifting amplifier in the appendix will aid in following the analysis).

With an amplitude modulated signal input of the form

$$E \cos \omega_d t \cos \omega_c t$$

or, equally well

$$E/2 \cos(\omega_c + \omega_d)t + E/2 \cos(\omega_c - \omega_d)t$$

the output of the upper parallel T (390 cps null point) is

$$j\, K_1/2 \cos(\omega_c + \omega_d)t \pm j\, K_2/2 \cos(\omega_c - \omega_d)t$$

and the output of the lower parallel T (410 cps null point)

$$\pm j\, K_2/2 \cos(\omega_c + \omega_d)t + j\, K_1/2 \cos(\omega_c - \omega_d)t$$

Assuming that K_2 is very small compared to K_1 due to the flatness of the notch (See the parallel T frequency response in the appendix.), then K_2 may be neglected. Neglecting K_2, the output of the two filters becomes

$$j\, K_1/2 \cos(\omega_c + \omega_d)t + j\, K_1/2 \cos(\omega_c - \omega_d)t$$

Applying this signal to the phase-shifting amplifier, the output of the

amplifier is (90° positive phase-shift in the amplifier)

$$-\frac{K_1}{2}\cos(\omega_c+\omega_d)t \quad -\frac{K_1}{2}\cos(\omega_c-\omega_d)t$$

The bypassed signal equals

$$\frac{E}{2}\cos(\omega_c+\omega_d)t \quad +\frac{E}{2}\cos(\omega_c-\omega_d)t$$

and adding the amplifier output to the bypassed signal, the resultant signal is

$$\left(\frac{E}{2}-\frac{K_1}{2}\right)\cos(\omega_c+\omega_d)t \quad +\left(\frac{E}{2}-\frac{K_1}{2}\right)\cos(\omega_c-\omega_d)t$$

which equals

$$(E-K_1)\cos\omega_d t\,\cos\omega_c t$$

since $K_1 = f(\omega_d) \equiv K\omega_d$, this can be expressed as

$$(E-K\omega_d)\cos\omega_d t\,\cos\omega_c t$$

and, as previously indicated, this is the desired result.

After the compensator was constructed, the frequency response was obtained using the same test circuit that was used for the phase-lead compensator; however, the response of the compensator did not meet expectations. Only a negligible reduction in signal amplitude with frequency was noted. This was felt to be due to the fact that the assumptions made in developing the compensator were not followed in practice; namely, the output of the filters for the first 10 cycles either side of

the null point could not be disregarded.

Although this portion of the thesis was apparently unsuccessful, I feel that the approach to the problem has merit and that it should be possible to develop a phase-lag compensator following the above procedure. The possibility of constructing an asymmetrical parallel T was considered, as noted before, but was not pursued further.

The obvious advantage to a designer of having phase-lead and phase-lag filters so closely related that one amplifier would serve for both and of being able to use either by merely selecting a toggle-switch position are apparent.

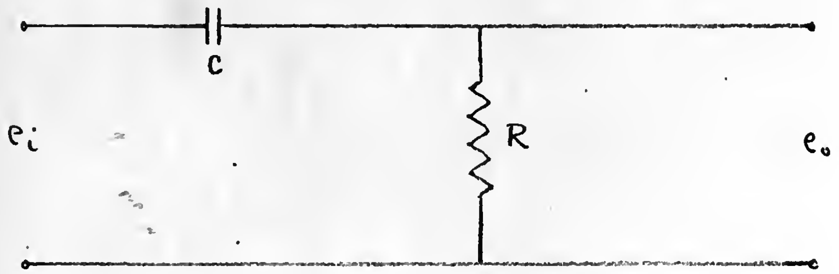
VII

BAND PASS FILTER COMPENSATOR

In DC compensation of servos, RC filters are commonly used as phase-lead and phase-lag filter compensators as was indicated in chapter II. The design of these filters is relatively simple and when the double T phase-lag compensator was not immediately successful, attention was given to the possibility of adapting these filters for use with an AC servo.

It will be recalled that the characteristic of an AC phase-lag compensator is a high output for a zero data frequency and decreasing signal strength with increasing data frequency. The basic idea is to obtain this response by cascading high and low pass filters having a common break point at the carrier frequency.

For the high pass filter



a transfer function is readily developed as follows:

$$e_o = \frac{e_i R}{R + 1/sC} \quad ; \quad \frac{e_o}{e_i} = \frac{sCR}{sCR + 1}$$

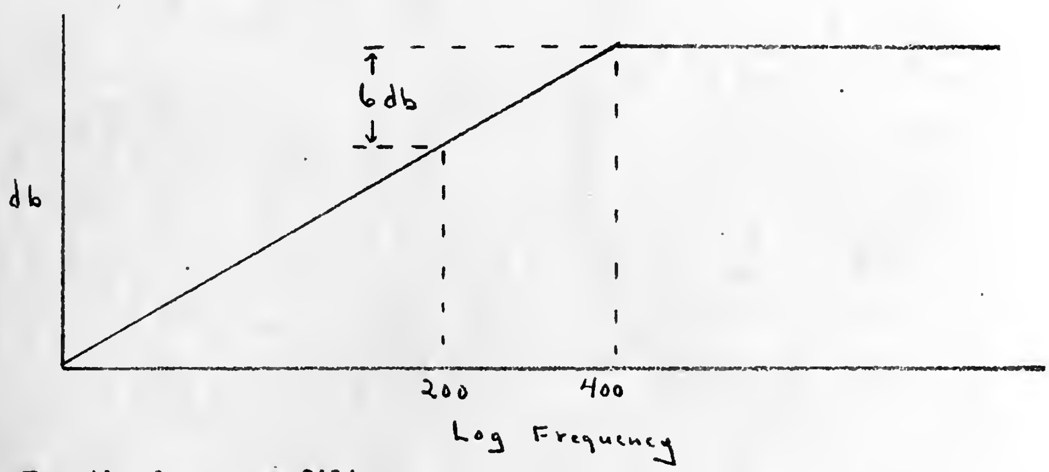
which may be written as

$$\frac{s\tau}{s\tau + 1} \qquad \text{where} \qquad \tau = RC$$

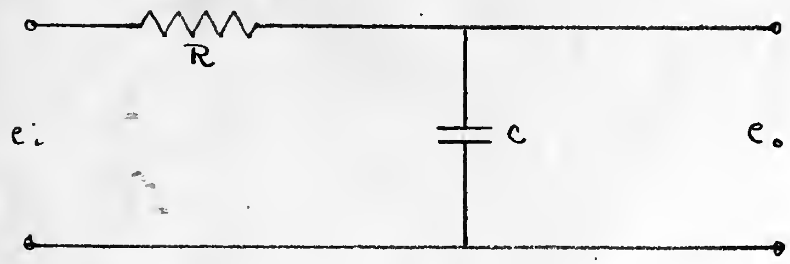
and for a break point located at 400 cps,

$$\tau = \frac{1}{\omega_c} = \frac{1}{2515} = 3.98 \times 10^{-4}$$

The asymptotic decibel-log ω plot of the foregoing filter is shown below.



For the low pass filter



the transfer function is

$$e_o = \frac{e_i \; 1/sC}{R + 1/sC} \quad ; \quad \frac{e_o}{e_i} = \frac{1/sC}{\frac{sCR+1}{sC}}$$

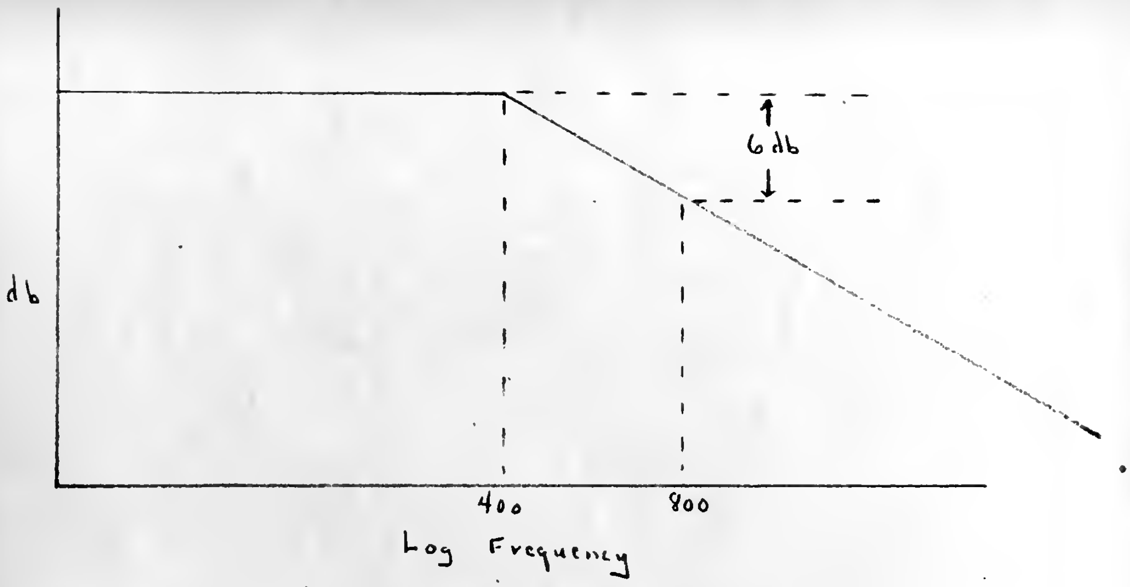
which equals

$$\frac{1}{s\tau + 1} \qquad \text{where, as before,} \quad \tau = RC$$

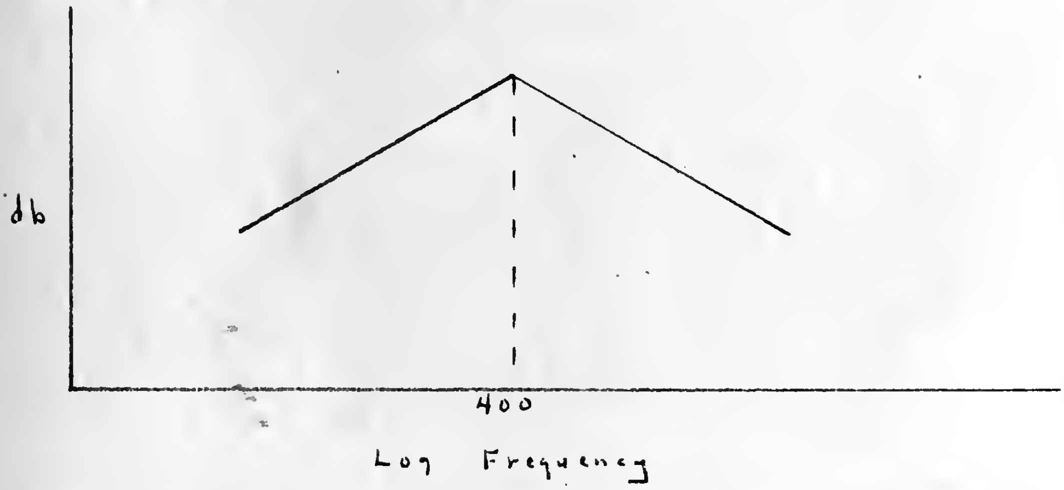
and

$$\tau = 1/\omega_c = 3.98 \times 10^{-4}$$

The above transfer function has the following asymptotic plot.



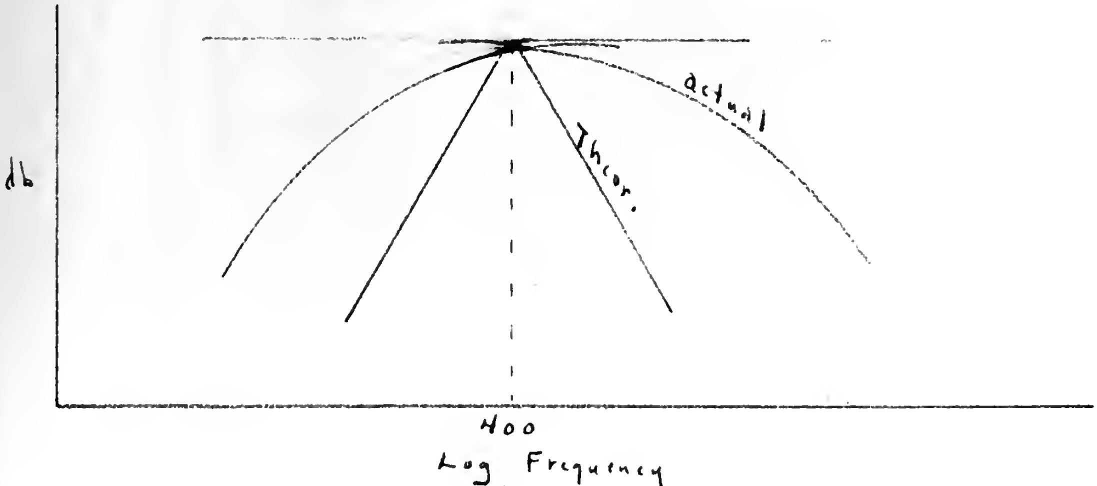
Cascading the filters results in the following plot.



From these plots it is evident that for two such filters in cascade, the attenuation per octave is six decibels. In view of the fact that an appreciable signal reduction must be achieved with data frequencies on the order of 20 cps, two such filters will be patently inadequate. However, if multiple stage filters are used, then an attenuation of 6K decibels per octave for each filter is theoretically possible, where K is the number of stages.

In order to verify the foregoing analysis, four stage filters were constructed; i. e., four stage high-pass and four stage low-pass filters. The frequency response for each filter separately gave an attenuation of 16 decibels per octave, which was felt to be acceptable. It was not possible to cascade the two filters directly, however, since the availability of components prevented the construction of eight stages with an impedance ratio of ten per stage. In order to match the output of the high-pass filter to the input of the low-pass, a vacuum tube amplifier was used as an isolation stage between filters. The complete circuit diagram of the filter is shown in the appendix.

The poor response of the compensator (about 6 decibels per octave) led to variations in the coupling circuit—e.g., a cathode follower in place of the amplifier, but with no change in the response. The poor response is attributed to the fact that the break point for the filters is not sharp, as is indicated in the asymptotic plots, but is much more gradual. This, of course, was known and the task of building the compensator was undertaken to determine the degree of variation. The preceding can be shown more effectively by consideration of the actual decibel-log ω plot for the compensator.



In the light of this response, it is felt that any reasonable increase in the number of stages per filter would have a negligible effect on the overall response. Accordingly, no further work was done on this aspect of compensation.

VIII

DISCUSSION, CONCLUSIONS, AND RECOMMENDATIONS

1. Discussion

In discussing the results of any experimental work it is well to point out the limitations imposed by the equipment itself and by the measuring devices used to obtain the data.

As regards the equipment, great care was taken to insure that the amplifiers were linear over the range of input signals applied and that the fit of the gearing used was such as to give a minimum of backlash and yet not bind at any point. The use of gearing, however, almost inevitably results in some non-linearity. Moreover, the two-phase induction motor is a non-linear device, contrary to the transfer function used in chapter III. At best, the two-phase motor approaches linearity only over a limited operating range. For a more complete discussion of the motor, reference is made to the literature.

The result of these non-linearities is apparent in the Bode Diagrams for the system. In the diagrams for the uncompensated and for the DC compensated system the solid lines represent the linear transfer function for a system with approximately the same magnitude of gain. The departure of the experimental points from these lines is indicative of the degree of non-linearity present in the system.

It has often been said that in taking experimental data, the selection of the measuring devices employed depends almost entirely on the degree of accuracy required in the particular application. And in obtaining frequency responses for any particular servomechanism component

or system, absolute magnitude levels are not so important as relative magnitude levels. This makes the two channel Brush Recorder well adapted to servomechanism work. Most of the data taken in this thesis were recorded on the Brush Recorder and the curves not directly obtainable from the recorder were developed from recorder data by the use of auxiliary graphical procedures. Magnitude values on the recorder may be read to accuracies within about two percent of the maximum value of the input signal, which is well within the requirements for frequency response measurements. Phase differences between the input signals, though not capable of as precise a measurement as magnitude differences, are still within the necessary requirements. Phase difference can be read with an accuracy varying between three and ten degrees, depending on the magnitude of the difference.

Although this thesis has been entitled "AC Compensation of AC Servomechanisms", it has by no means encompassed the entire field of AC compensation. On the contrary, it has considered only a very limited aspect of compensation; namely, what has been termed "Series AC Compensation" by some authors. And series compensator possibilities have been far from exhausted. Some of the factors which governed the selection of the particular compensators studied were: (1) economy, (2) ease of design and manufacture, and (3) a design need for the compensator. By the last is meant the ability of the compensator to be adjusted so that it may be used in a large number of applications without the necessity for redesign for each application.

2. Conclusions

As a result of this investigation, it has been definitely determined:

a. That the parallel T compensator used in commercial equipment has very little effect on performance and essentially produces no beneficial results. This can readily be seen in the frequency response of the commercial compensator which appears in the appendix. At a data frequency of zero cycles per second, the output of the compensator is 0.14 volts for a one volt input; while at a data frequency of 16 cps, the output has increased 0.01 volts to a total of 0.15 volts. This is a negligible change in magnitude; moreover, the log w vs. magnitude and phase angle curve shows that this slight change in magnitude is accompanied by a change in phase angle of only ten degrees. The straight parallel T with a bypass section can therefore be eliminated as an adequate series compensator.

b. The parallel T can be made to supply adequate phase-lead compensation if the output of the filter is preamplified before being added to the bypassed signal. The difficult problem of adding the preamplifier output to the bypassed signal was satisfactorily solved during the course of this investigation. The effectiveness of this compensator can only be appreciated by examination of the frequency response curve and the log w vs. magnitude and phase angle curve in the appendix. The frequency response curve indicates that for zero data frequency the output of the compensator is one volt for a one volt input; while for a data frequency of 16 cps, the output is 1.7 volts, and for a data frequency of 21 cps the output voltage is exactly doubled. This is to be compared to the response of the straight parallel T compensator under (a) above. Furthermore, accompanying this change in magnitude is a change in phase angle of sixty-five degrees.

This compensator not only produces satisfactory results but is the best method of series phase-lead compensation which has been encountered

by the author.

c. The use of cascaded high-pass and low-pass filters as a phase-lag compensator is not practical because the attenuation of the signal as a function of the data frequency cannot be made sharp enough.

3. Recommendations

In making recommendations for future thesis topics, I have restricted myself to the particular field covered by this thesis; i. e., to series AC compensators.

A great deal of research has been done on the subject of phase-lead compensation and particularly on the use of the parallel T circuit as a compensator. But, to my knowledge, none of the compensators previously developed can compare to the effectiveness of the parallel T-preamplifier circuit. Since this thesis was concerned primarily with ideas and not with optimum results, a thesis topic which would be particularly appropriate to the Postgraduate School would be a study of the parallel T-preamplifier circuit with an end view of reducing component size and obtaining optimum response. This project, though more of an applied engineering problem than a research problem, seems to me to be of sufficient interest to the Navy as to make the topic acceptable.

In spite of the fact that I was unable to make the double T phase-lag compensator work successfully in the time available, I feel that it was primarily due to the fact that the output of the parallel T's was too high near the null point. (This, of course, is desirable in a phase-lead compensator). It is my belief that the proposed circuit will work; that it is merely a question of shaping the parallel T response until it approximates the assumptions made in the development of the compensator. At least one of

the items which has an effect on the shape of the frequency response is the relationship between circuit parameters. Some work has been done in this regard but with a different end in view.

Still another possibility of approximating the desired response lies in the use of asymmetrical parallel T's. These circuits have also been investigated in some detail, and before any actual experimental work is attempted it would be advantageous to make a careful study of the literature available on the subject.

The experimental verification of an altogether different type of AC compensator, which has been proposed by Mitchel and Guidal of the Naval Electronics Laboratory, might be a suitable thesis topic. The compensator proposed by Mitchel and Guidal appears in a report prepared for the Bureau of Ships and entitled "Stabilizing Systems for AC Servos". The report may be obtained from ASTIA, where it is listed under ATI number 83123. The report was read and briefly considered in that the design formulae furnished by the authors were utilized to design a phase-lag compensator for the basic test servomechanism. That is, the design formulae were followed as far as practicable. The limiting factor in the formulae is the Q of the coils used in the compensator. None of the coils available had a sufficiently high Q. This difficulty may be overcome in particular instances by the manufacture of high Q coils or by synthetically raising the Q of the coils available by the use of vacuum tube circuits. This particular investigation may be well worth while in view of the amount of work done by Mitchel and Guidal. If the vacuum tube approach were successful, it would remove a big restriction on the design formulae.

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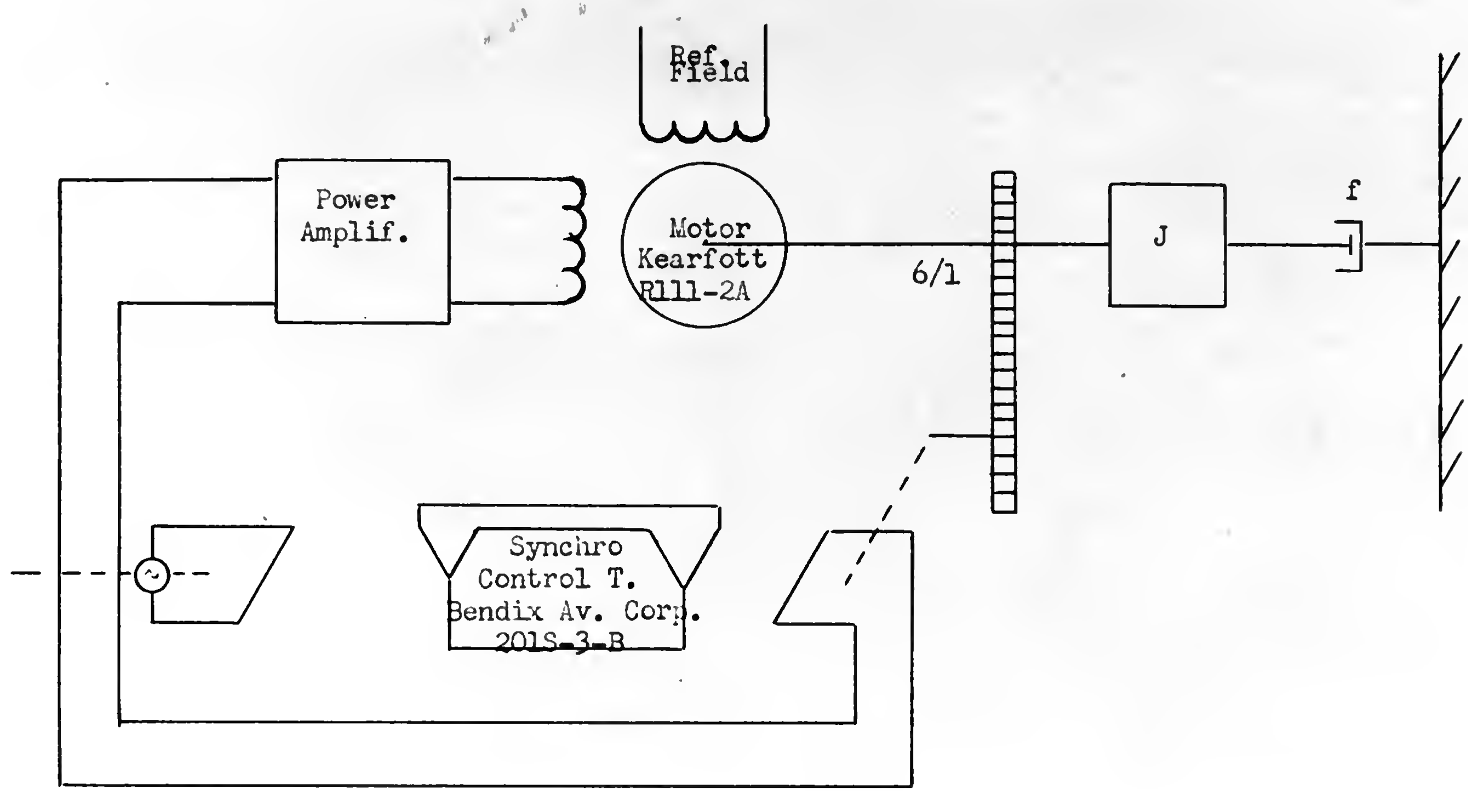
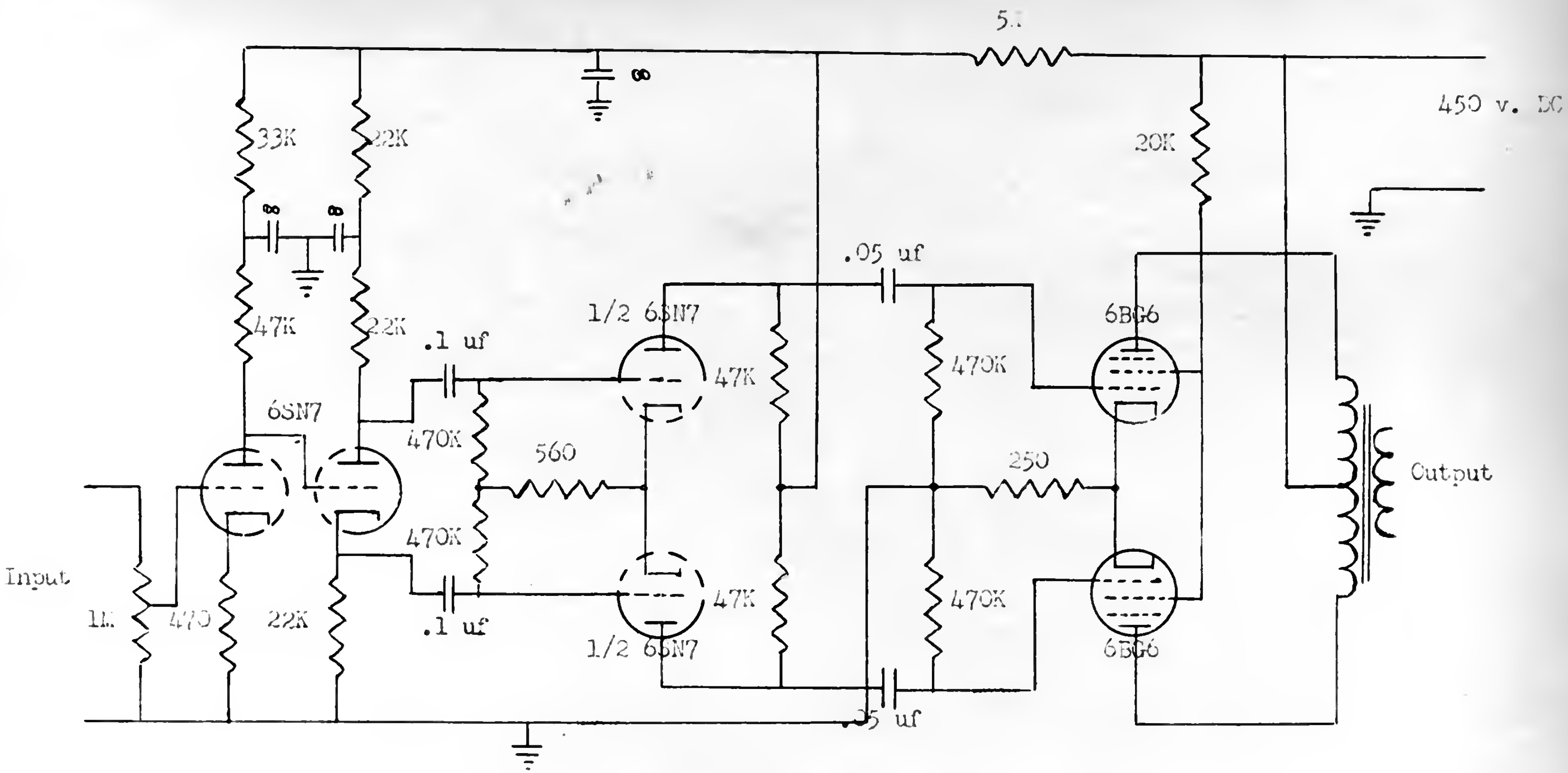
2.	James, H. M., Nichols, N. B., and Phillips, R. S.	Theory of Servomechanisms
3.	Lauer, H., Lesnick, R., and Matson, L. E.	Servomechanism Fundamentals
4.	Members of the Staff of the Department of Electrical Engineering, Massachusetts Institute of Technology	Applied Electronics
5.	Ryder, J. D.	Electronic Engineering Principles
6.	Thaler, G. J.	Elements of Servomechanism Theory (Manuscript)
7.	Thaler, G. J. and Brown, R. G.	Servomechanism Analysis

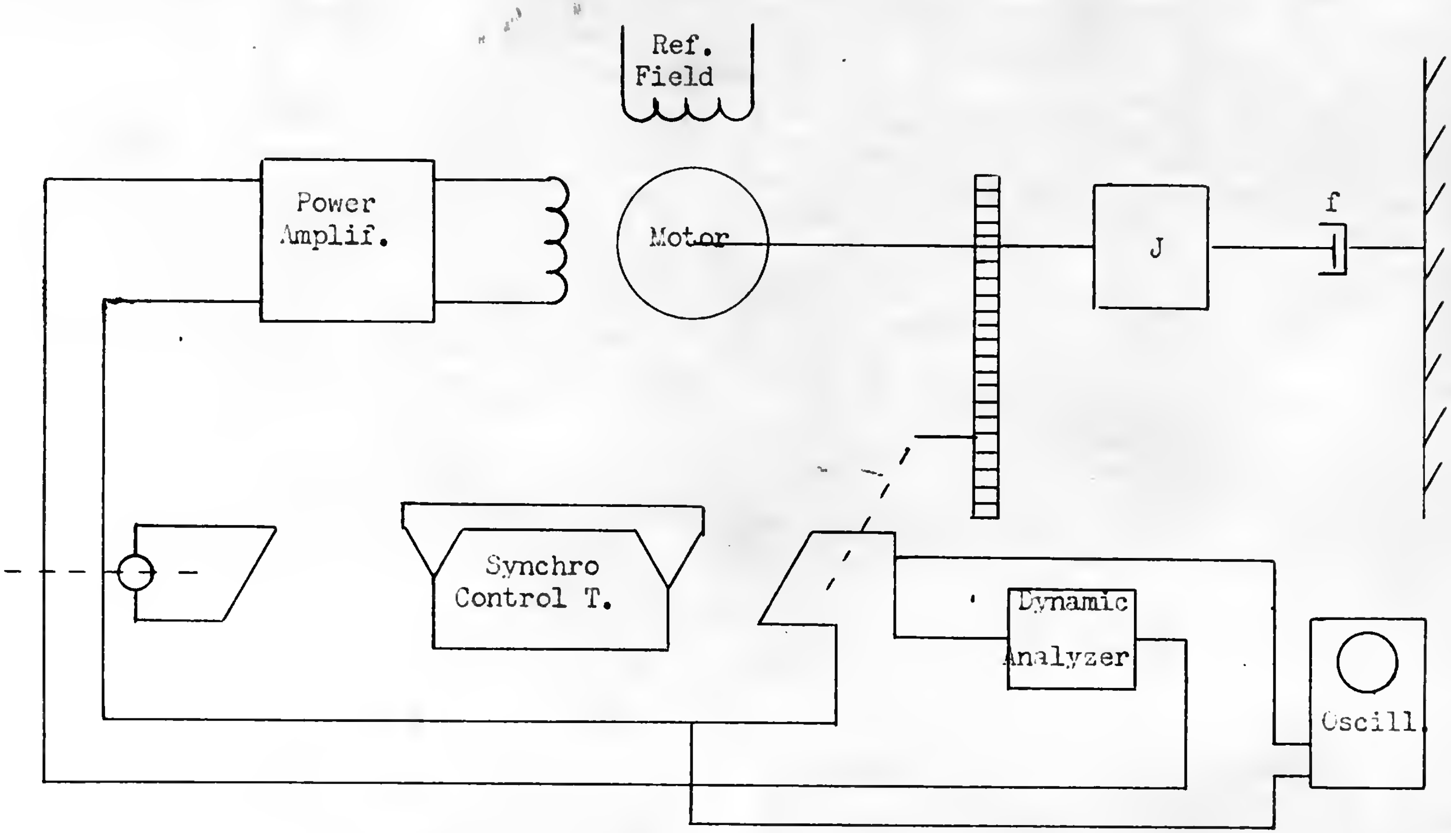
APPENDIX



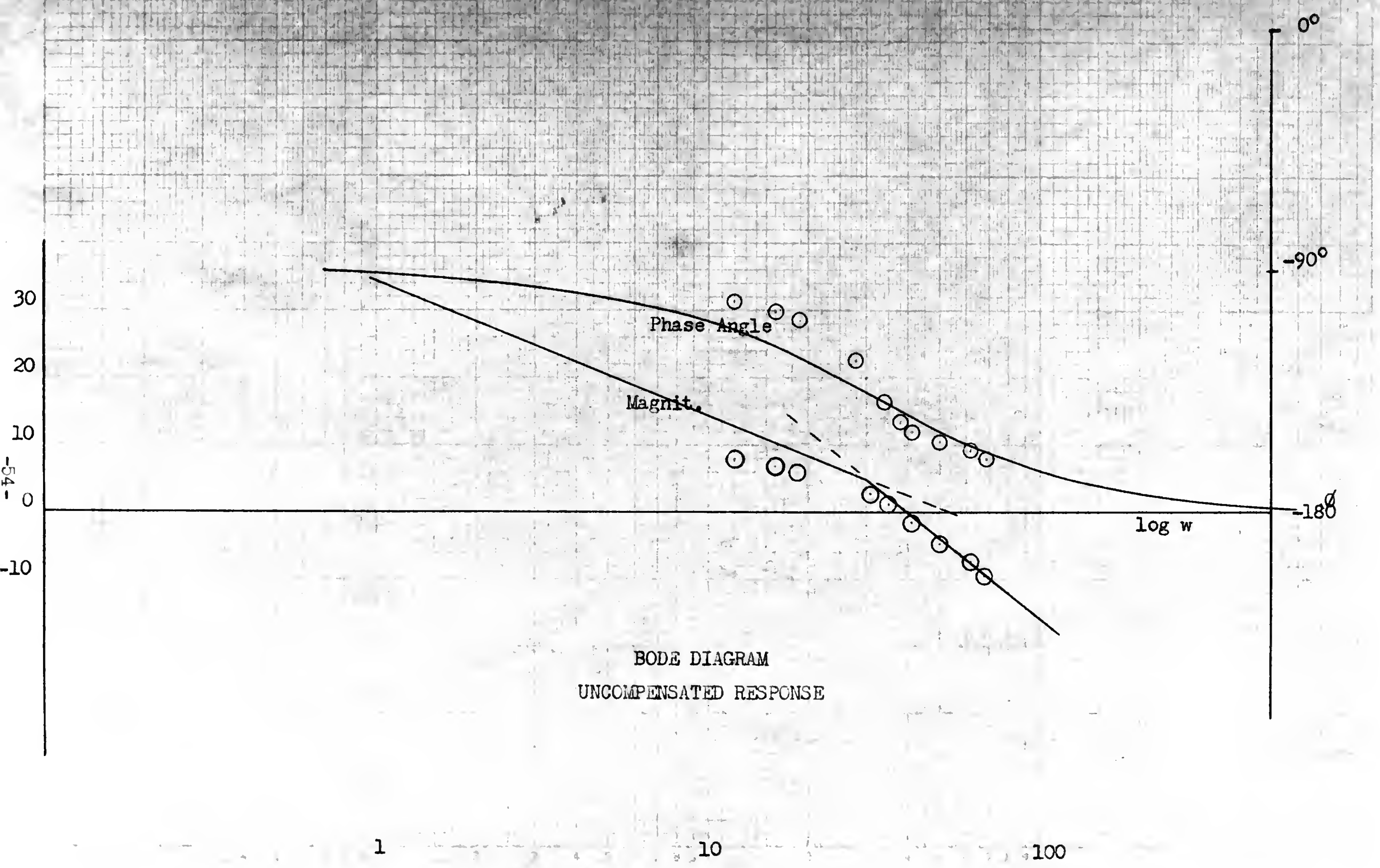
SIMPLIFIED SCHEMATIC OF BASIC SERVO



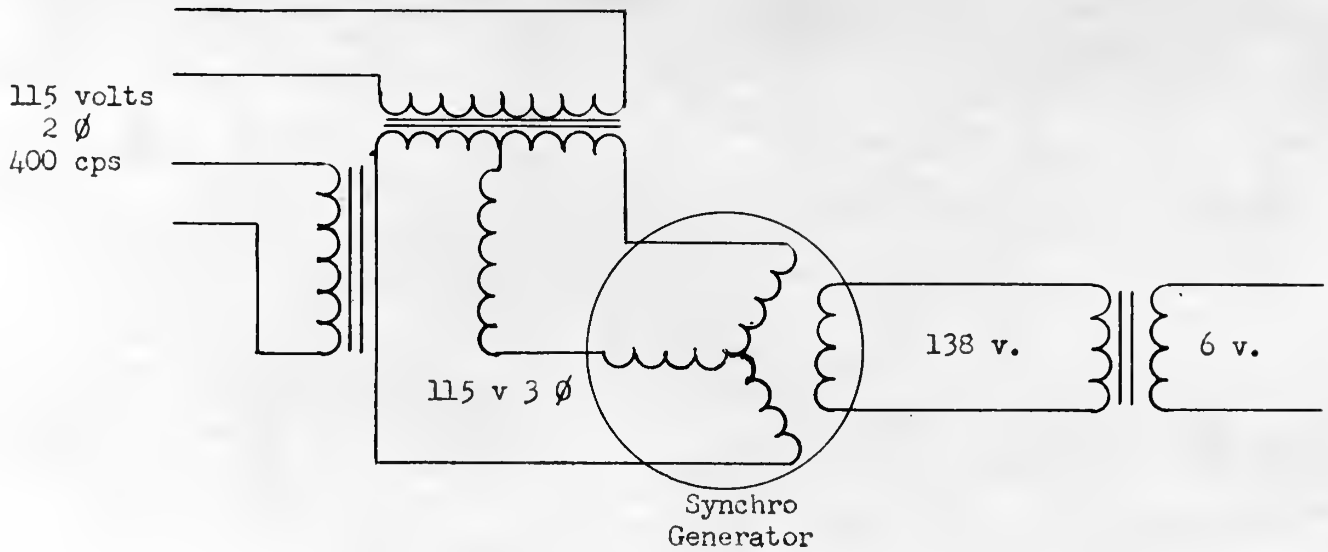
POWER AMPLIFIER



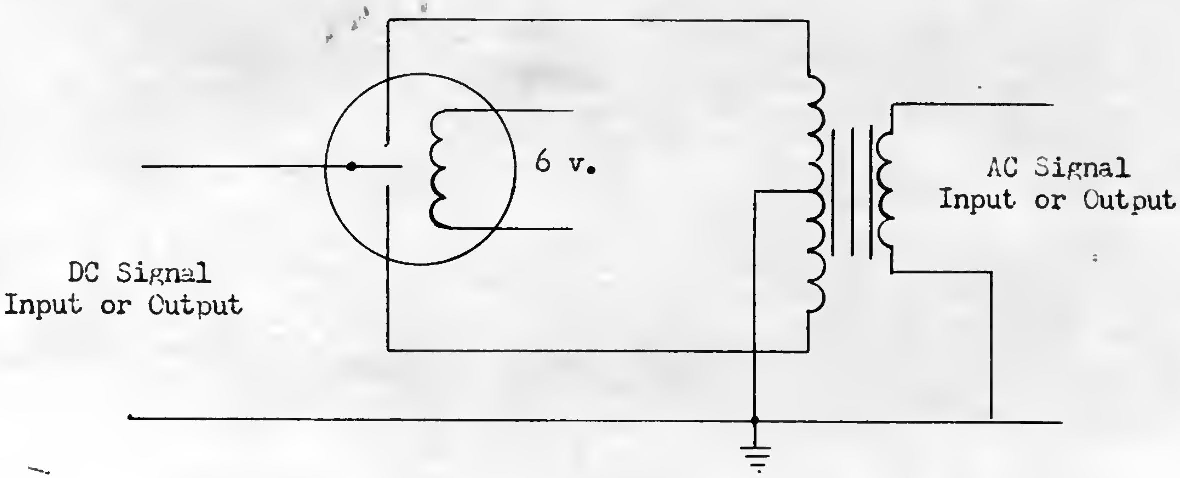
SIMPLIFIED SCHEMATIC FOR UNCOMPENSATED
FREQUENCY RESPONSE



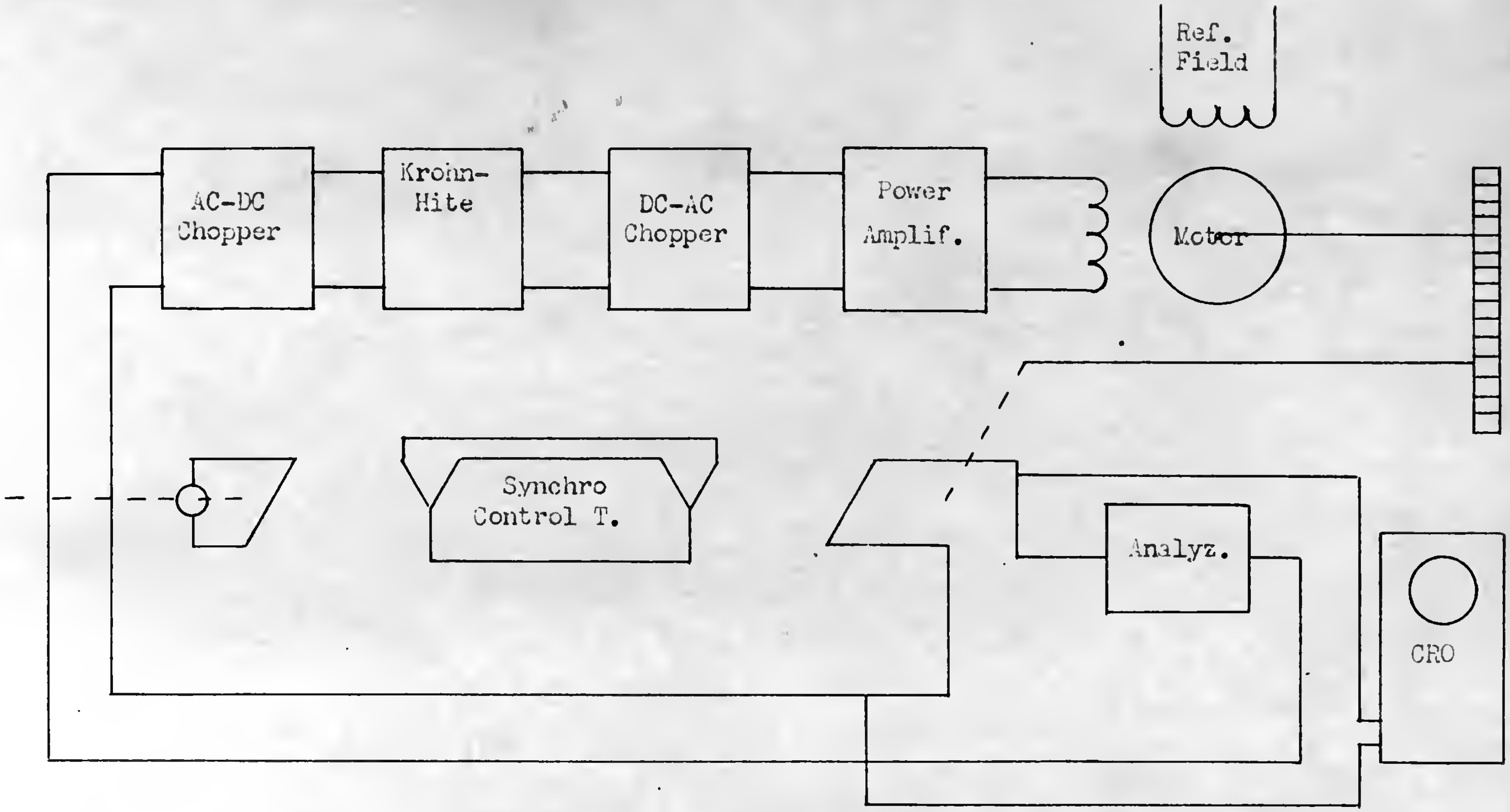
BODE DIAGRAM
UNCOMPENSATED RESPONSE



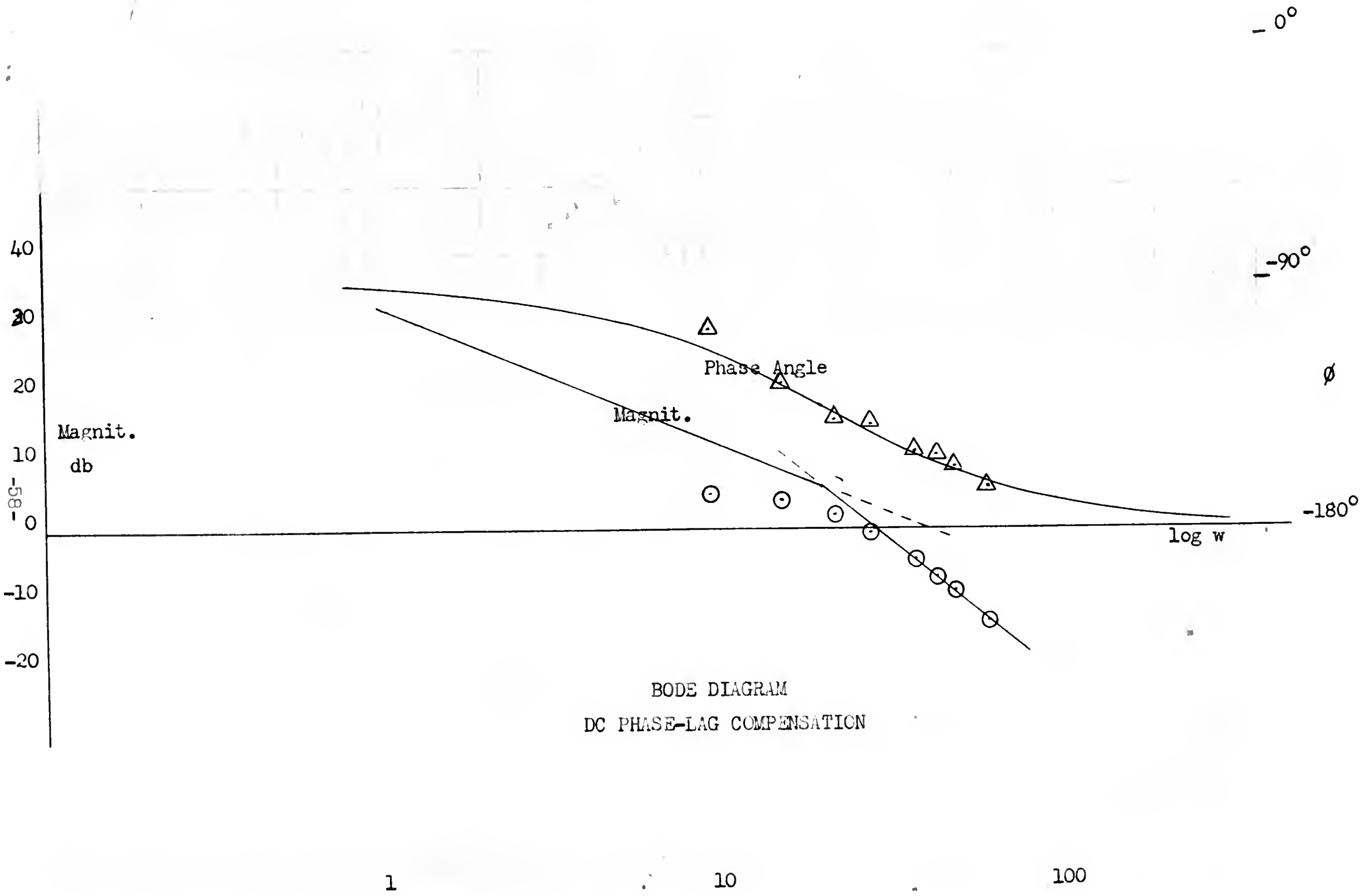
SCOTT TRANSFORMER AND PHASE SHIFTER



CHOPPER CIRCUIT

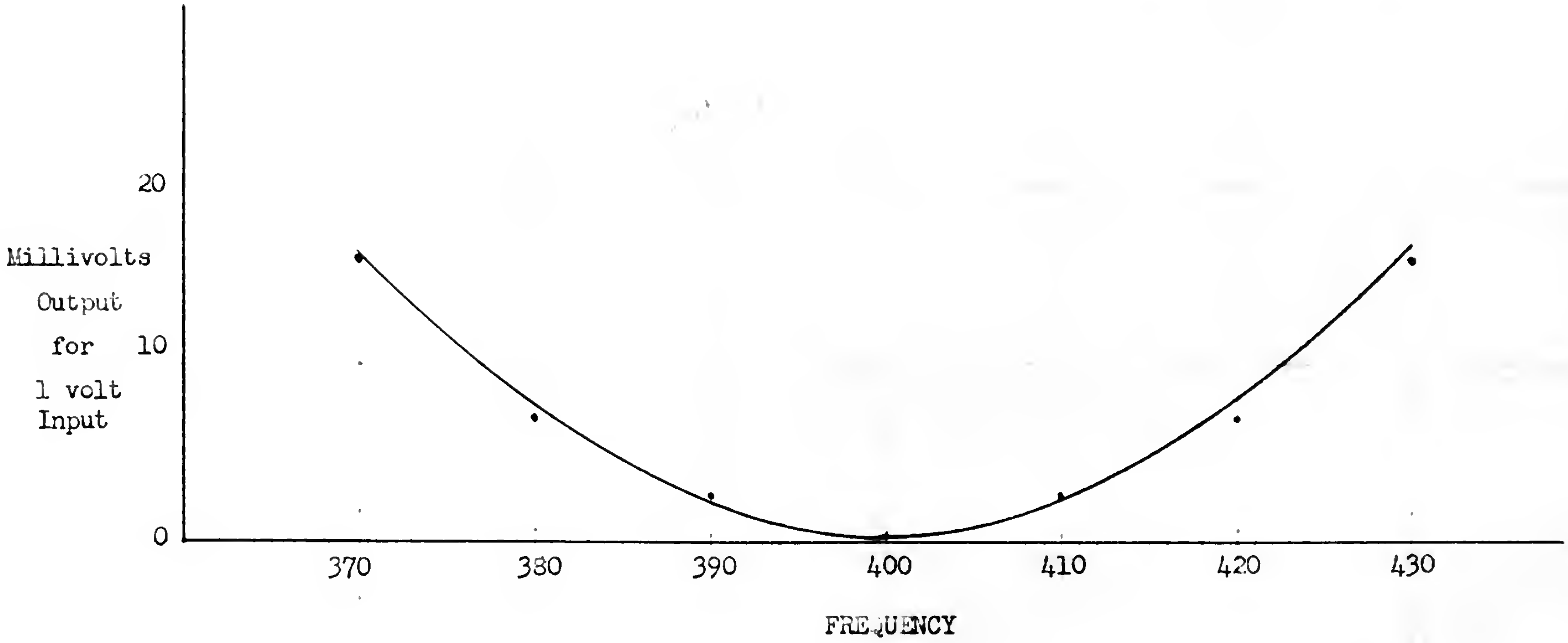


SIMPLIFIED SCHEMATIC OF TEST CIRCUIT
FOR DC COMPENSATED SYSTEM

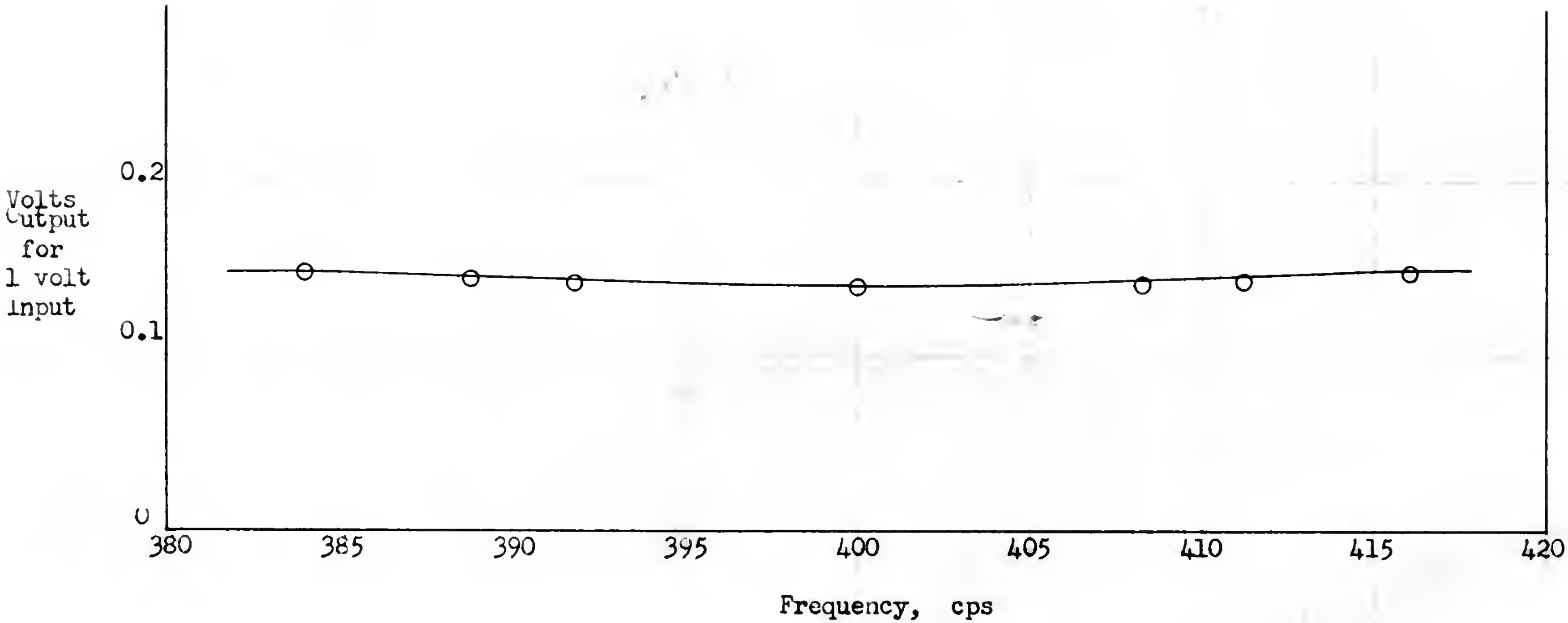


BODE DIAGRAM

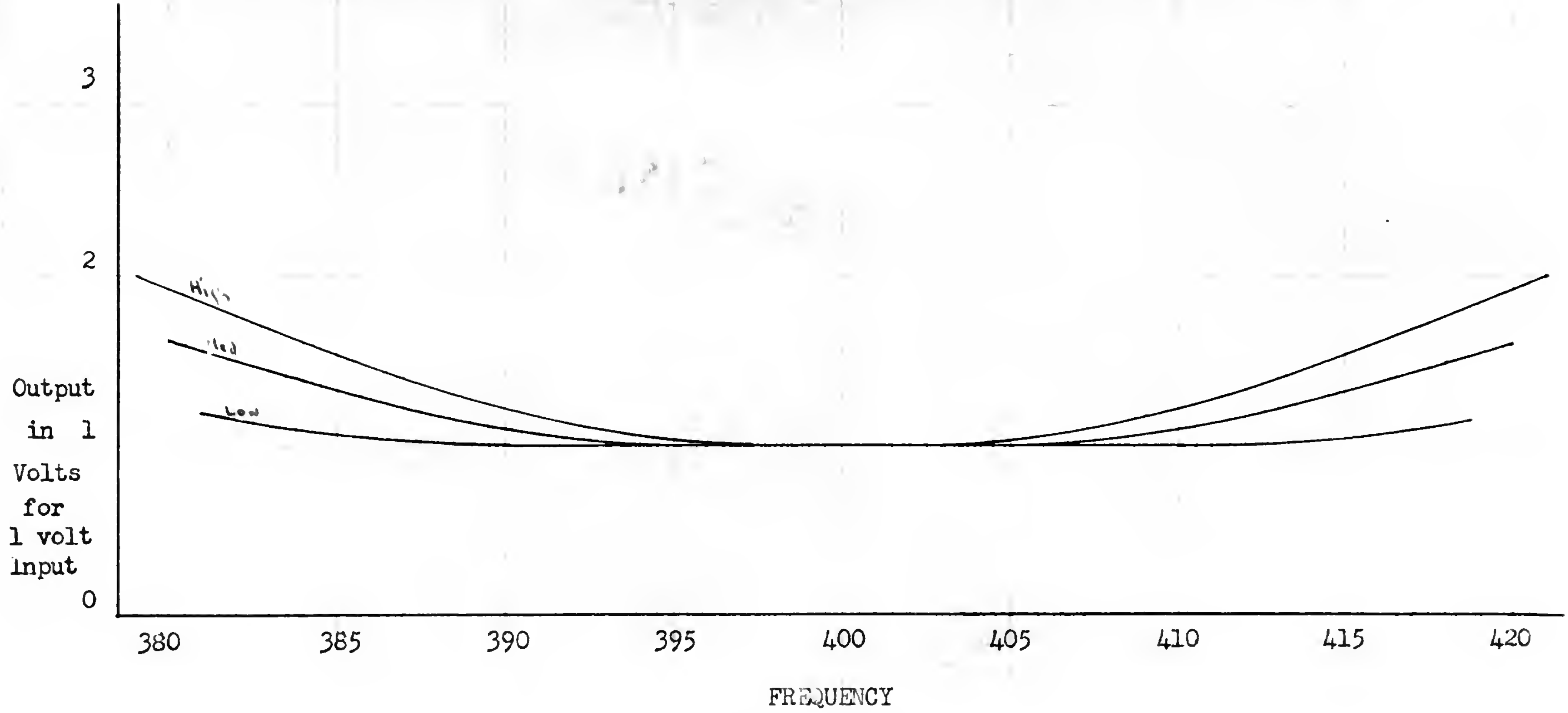
DC PHASE-LAG COMPENSATION



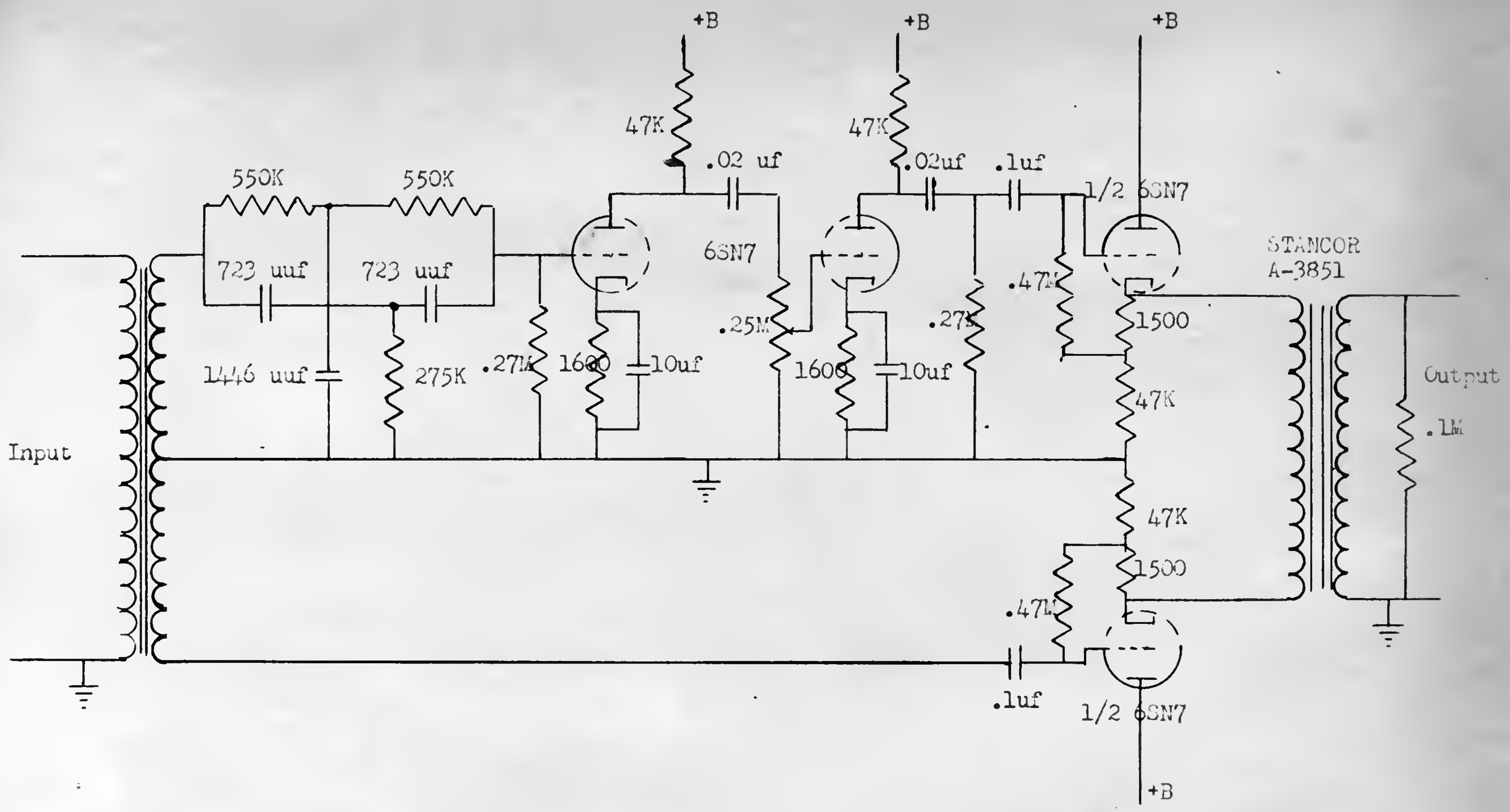
FREQUENCY RESPONSE OF COMMERCIAL
PARALLEL T



FREQUENCY RESPONSE OF COMMERCIAL COMPENSATOR

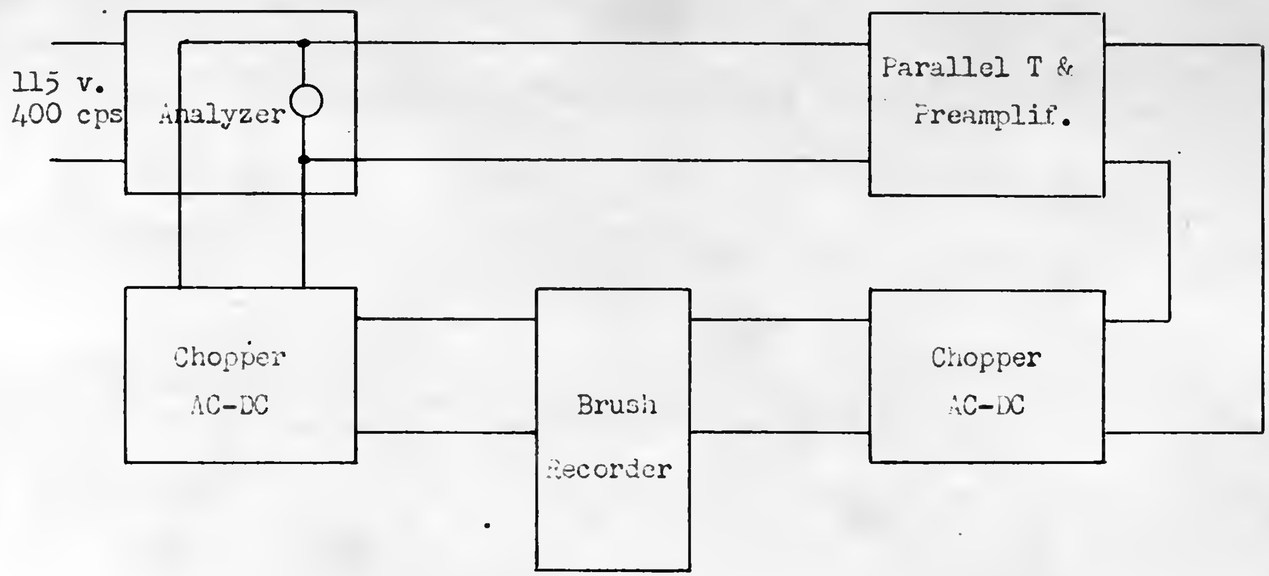


FREQUENCY RESPONSE OF PARALLEL T FILTER
AND PREAMPLIFIER



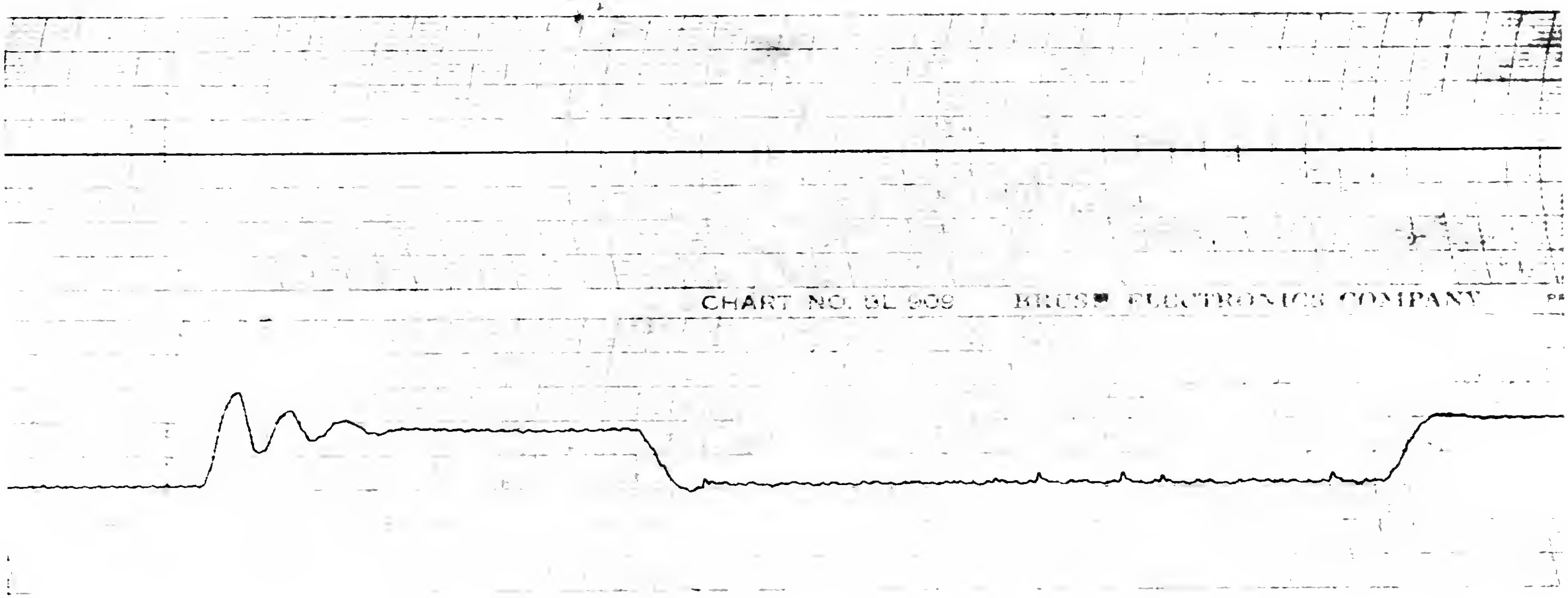
CIRCUIT DIAGRAM OF

PARALLEL T AND PREAMPLIFIER



SIMPLIFIED SCHEMATIC OF CIRCUIT USED TO

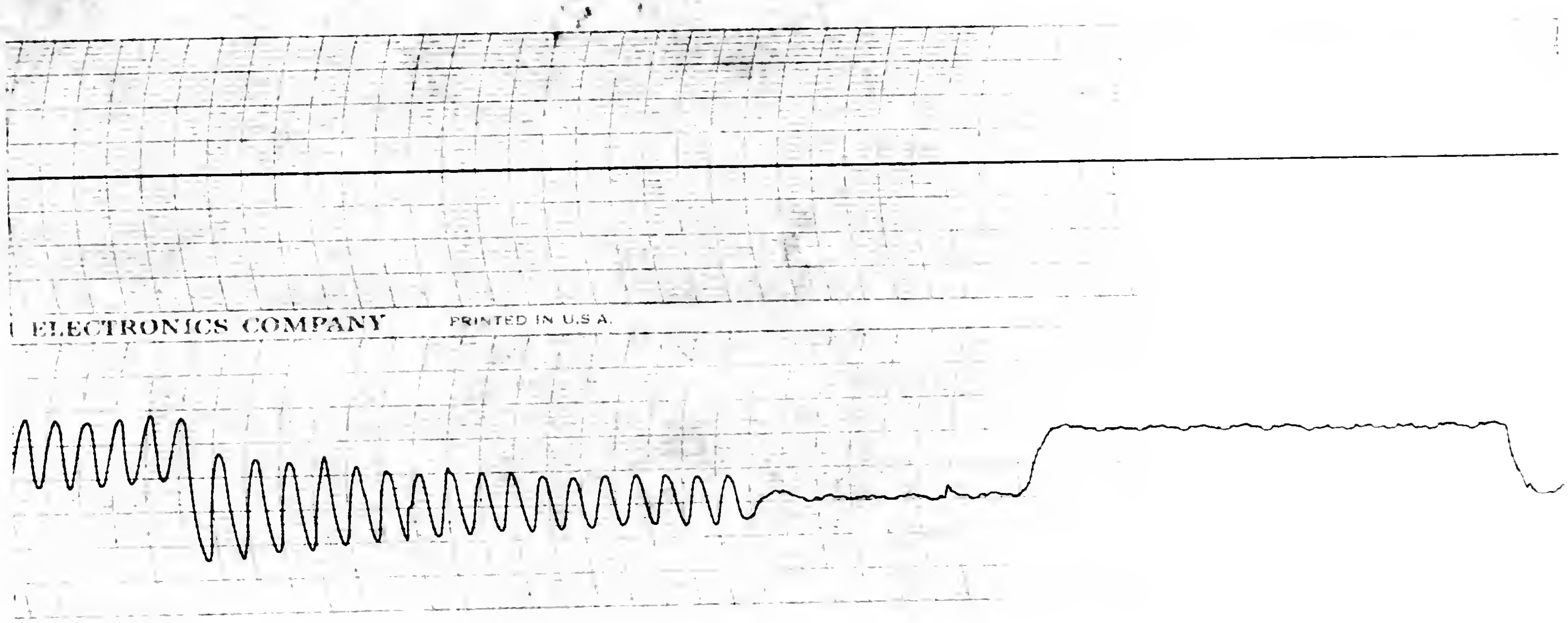
OBTAIN FREQUENCY RESPONSE OF PARALLEL T & PREAMPLIFIER



COMPARISON OF UNCOMPENSATED AND COMPENSATED

TRANSIENT RESPONSE (PARALLEL T AND PREAMPLIFIER COMPENSATOR)

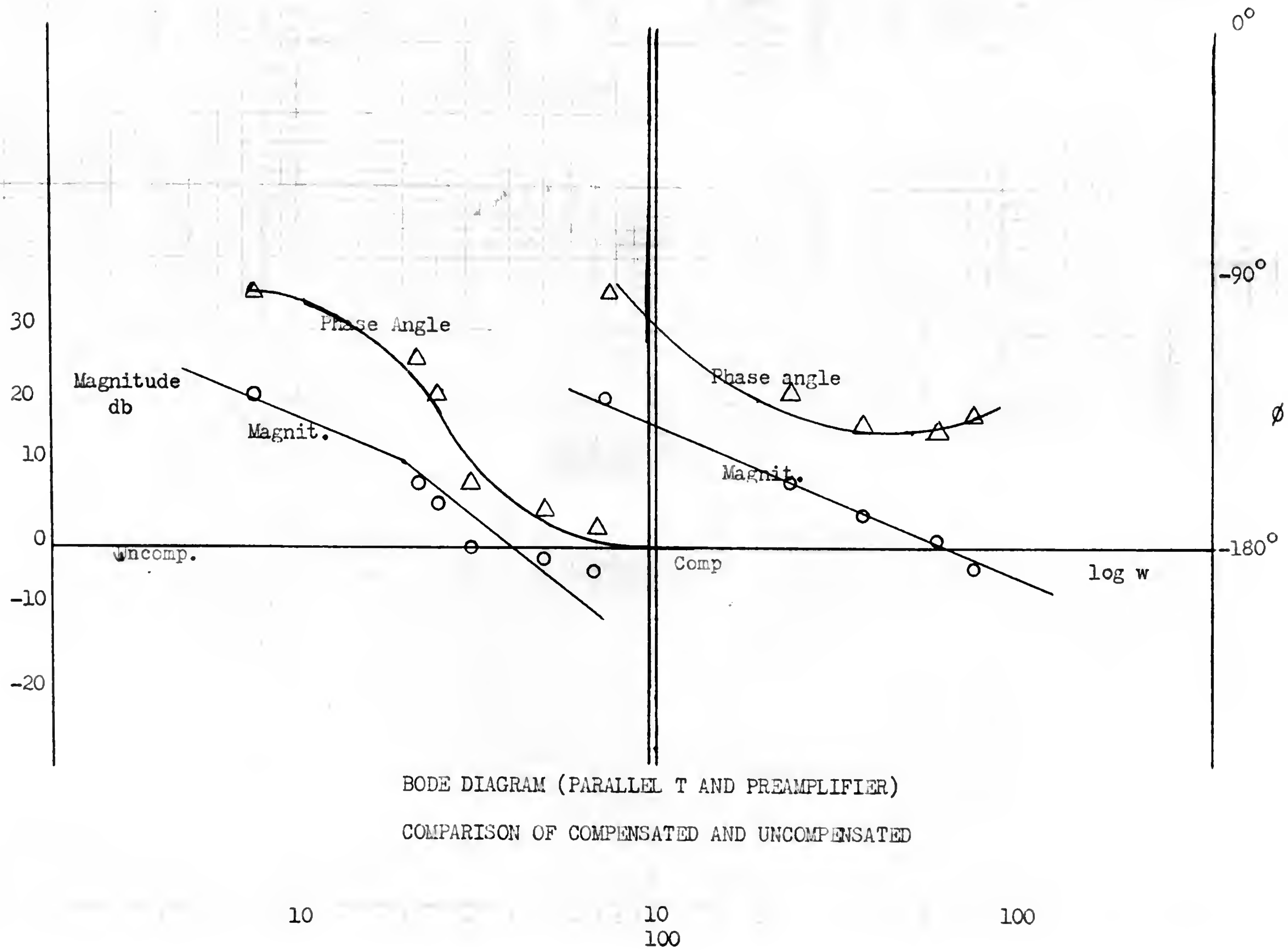
(INTERMEDIATE POWER AMPLIFIER GAIN)



COMPARISON OF UNCOMPENSATED AND COMPENSATED

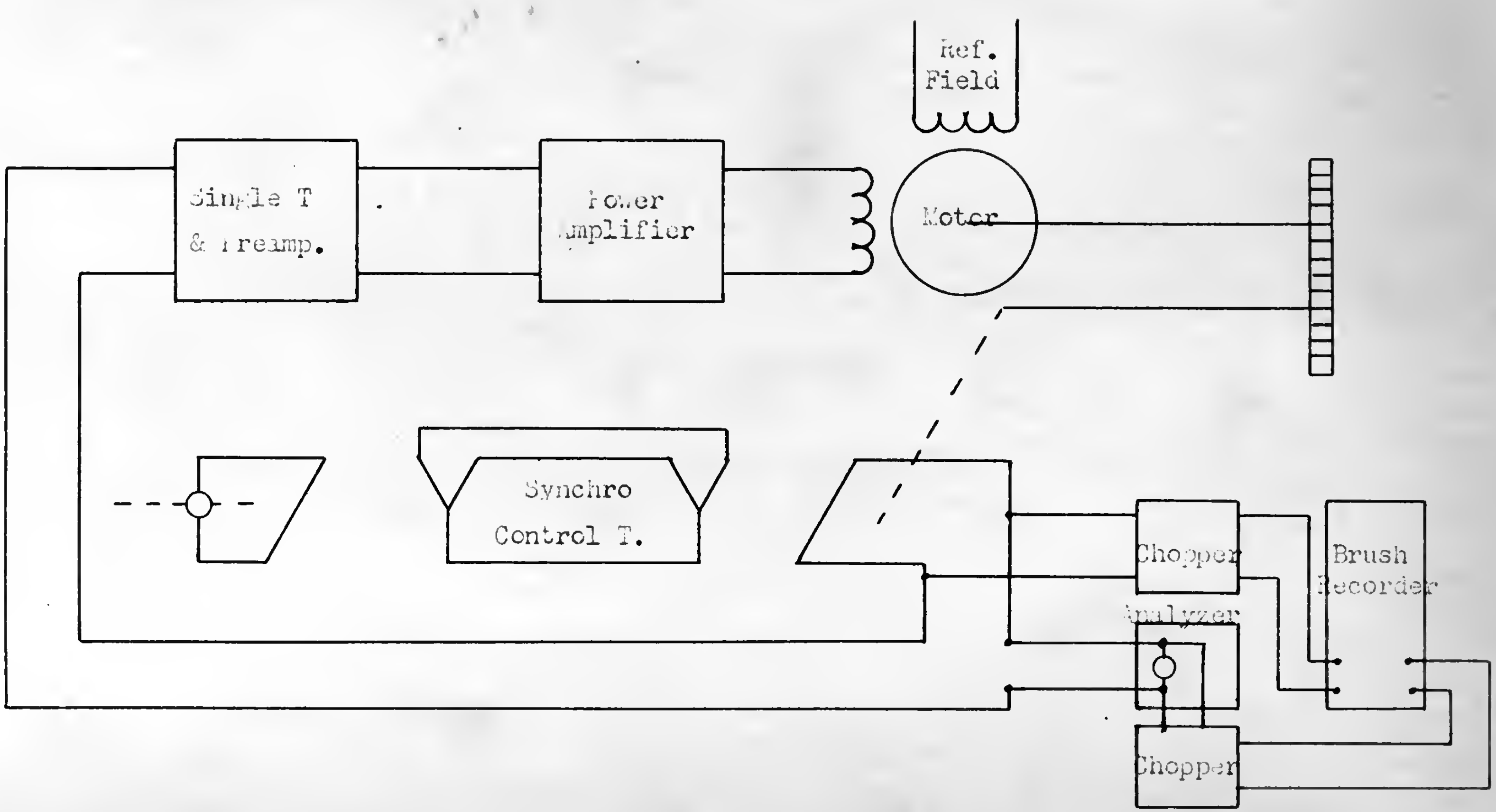
TRANSIENT RESPONSE (PARALLEL T AND PREAMPLIFIER COMPENSATOR)

(HIGH POWER AMPLIFIER GAIN)



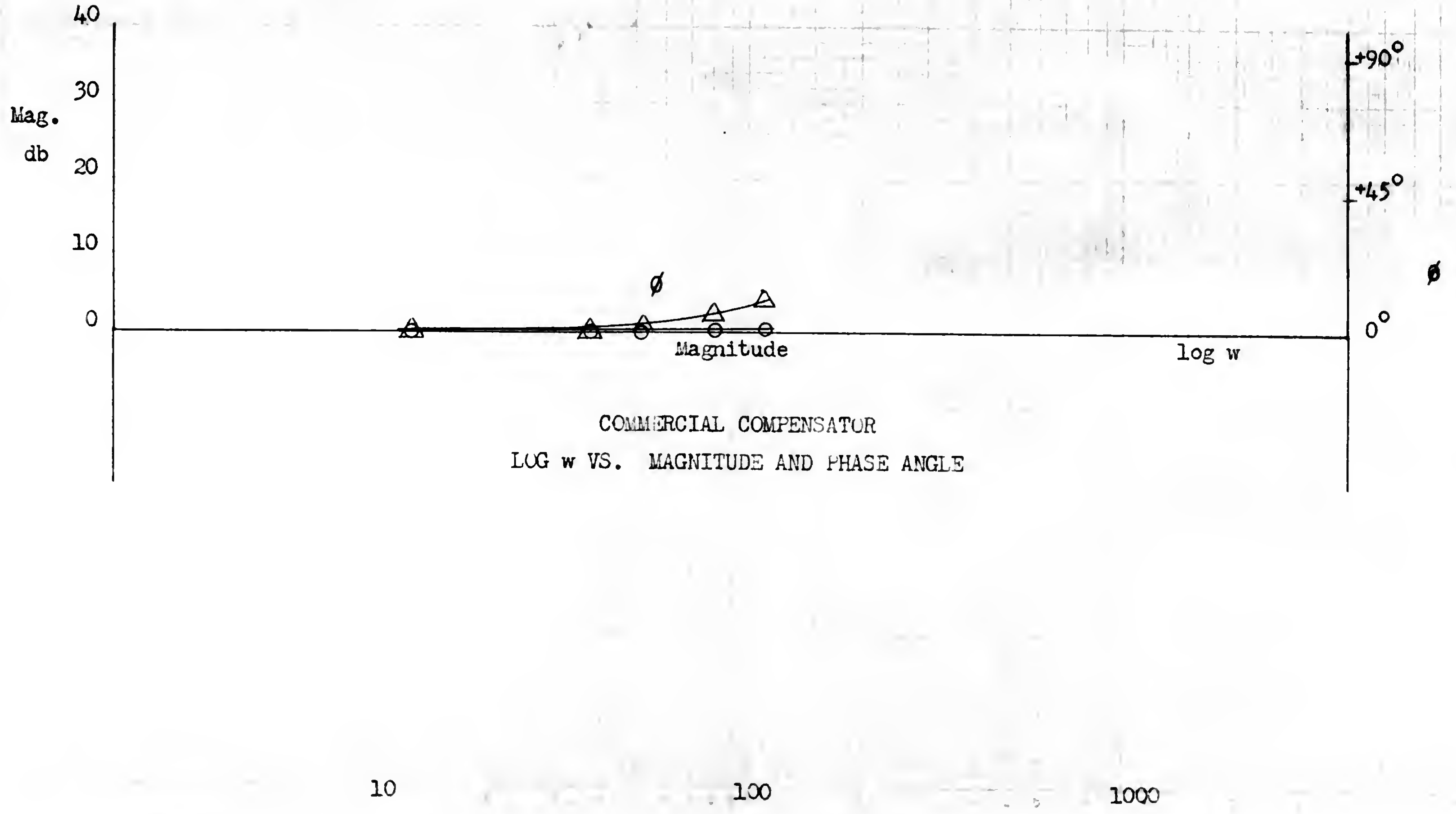
BODE DIAGRAM (PARALLEL T AND PREAMPLIFIER)

COMPARISON OF COMPENSATED AND UNCOMPENSATED



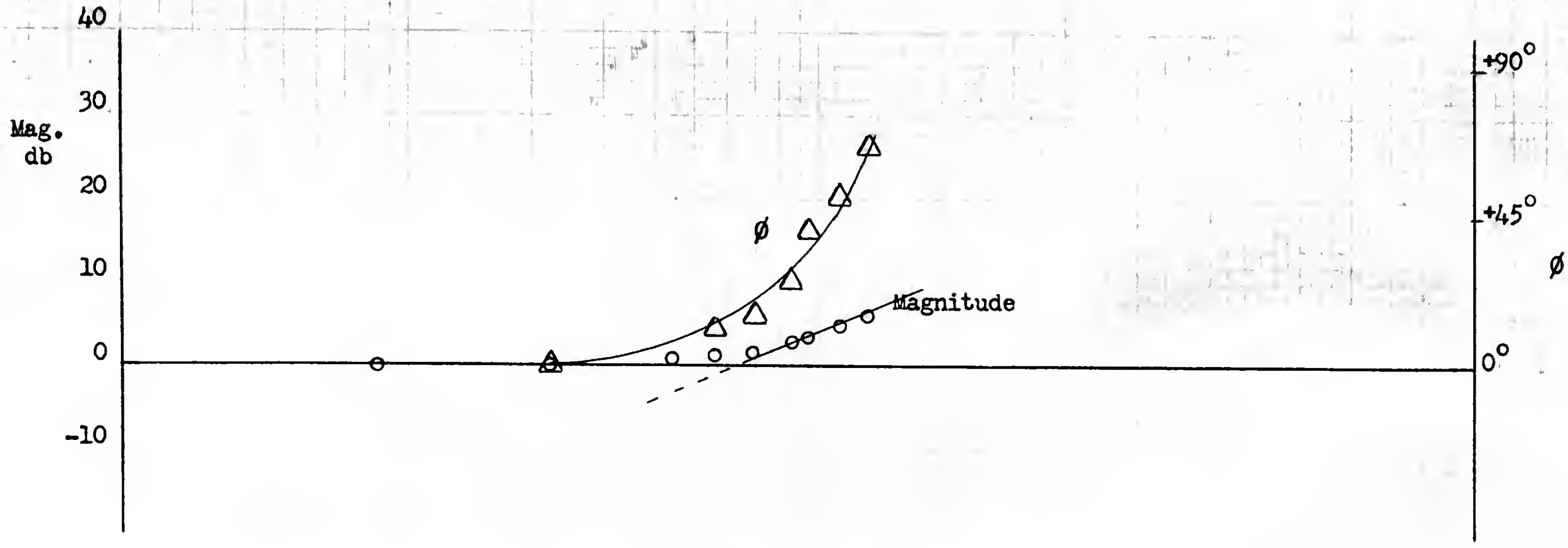
SIMPLIFIED SCHEMATIC OF TEST CIRCUIT FOR

PARALLEL T AND PREAMPLIFIER COMPENSATED SYSTEM



COMMERCIAL COMPENSATOR

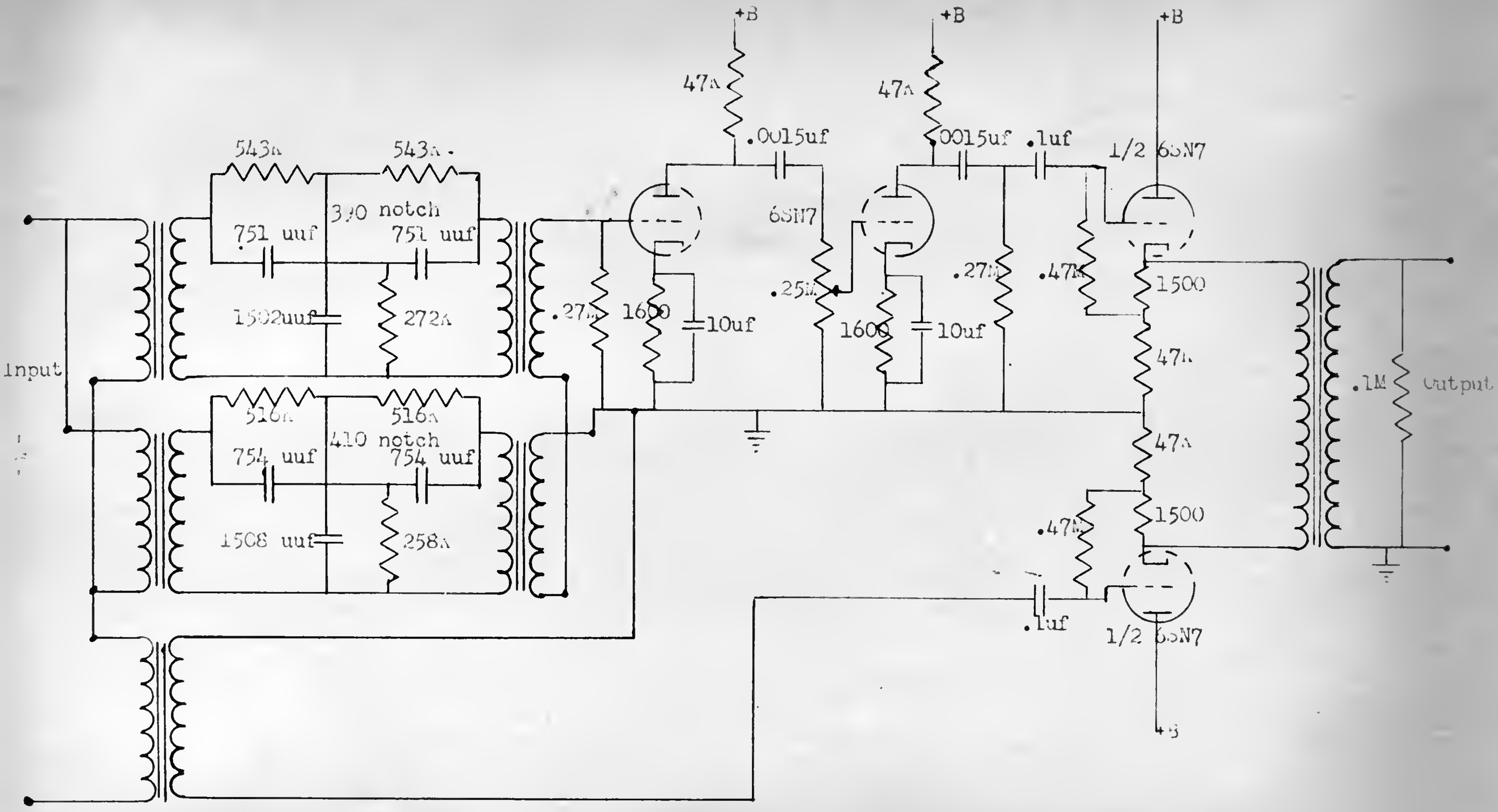
LOG w VS. MAGNITUDE AND PHASE ANGLE



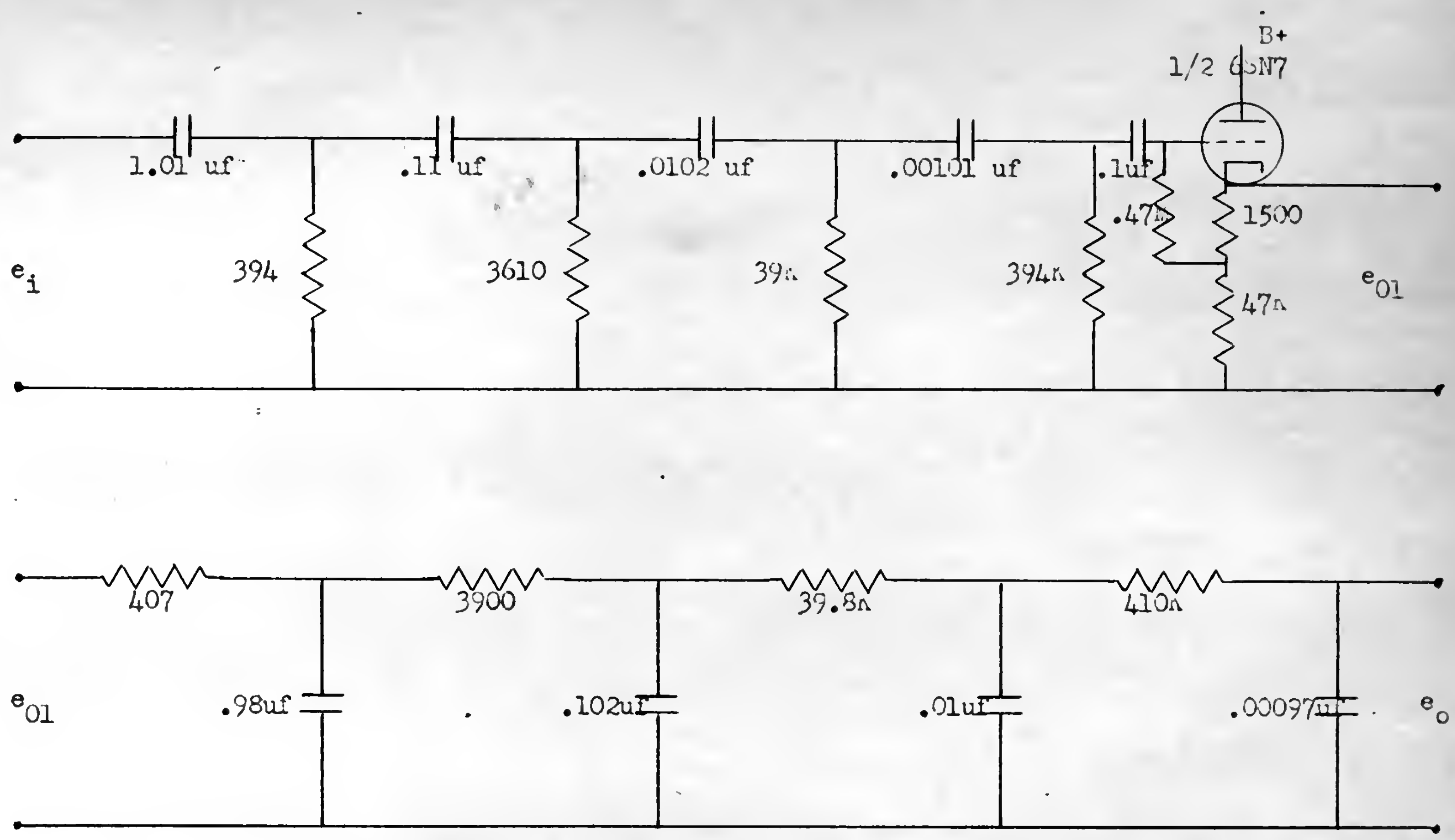
PARALLEL T AND PREAMPLIFIER

LOG w VS. MAGNITUDE AND PHASE ANGLE

10 100 1000



CIRCUIT DIAGRAM
PHASE LAG COMPENSATOR



CIRCUIT DIAGRAM

BAND PASS FILTER COMPENSATOR

BINDERY
DISPLAY
MAY 18
AP 16 56
AG 20 58
JL 25 60
MR 23 61
4239
5068
9530
10923